

Linear and Non-Linear Modelling of Nigerian Crude Oil Prices

Abstract

To model Nigeria crude oil prices, this analysis compared univariate linear models to univariate nonlinear models. The data for this analysis was gathered from the Central Bank of Nigeria (CBN) Monthly Statistical Bulletin. The upward and downward movement in the series revealed by the time plot suggests that the series exhibit a regime-switching pattern: the cycle of expansion and contraction. At lag one, the Augmented Dickey-Fuller test was used to test for stationarity. For univariate linear ARIMA (p, d, q) and univariate non-linear MS-AR, seven models were estimated for the linear model and two for the non-linear model. The best model was chosen based on the criterion of least information criterion, AIC (2.006612), SC (2.156581), and the maximum log-likelihood of (-150.5480) for the crude oil prices were used to pick MS-AR (1) for the series. In analysing crude oil prices data, the MS-AR model proposed by Hamilton outperforms the linear autoregressive models proposed by Box- Jenkins. The model was used to predict the series' values over a one-year cycle (12 months).

Keyword: Crude oil Prices, linear models, non-linear models and forecasting.

Introduction

Oil and gas production and sales are Nigeria's primary sources of revenue. Oil was first uncovered in 1956 at Oloibiri in Bayelsa State, according to the National Petroleum Corporation (NNPC 2013). Shell D'Arcy, now Shell Petroleum Company, discovered oil after numerous exploration efforts that began in 1938.[5] Nigeria's abundance of oil gained her a reputation in the international market when it became an oil-producing country in 1958 when the first oil field started producing 5,100 barrels per day. Nigeria is now Africa's largest producer of oil, accounting for approximately 33-35 per cent of the continent's oil and gas capitals, the Organization of Petroleum Exporting Countries (OPEC) fifth-biggest exporter, and the United States' fifth-largest oil exporter. [14]

Crude oil has long been a major source of revenue in the Economic growth in Nigeria, and it is used in critical activities such as fuelling, transportation, and home heating. Nigeria's crude prices fluctuation is currently unknown, as an increase in crude oil price will increase the amount of revenue in the economy while also increasing the cost of goods. The nation's revenues follow the same pattern as oil prices rise but also fall. Because of the increased in cost of transportation, an increase in the price of oil may decrease the availability of other goods. The production and sales of crude oil have been on the decline for a while and then unexpectedly increased; this phase of increase and decrease (fluctuation) necessitates a process switch. The most widely used method for modelling time-series data is the Box - Jenkins' Autoregressive integrated moving average (ARIMA) model. When the patterns of the sequence under analysis are influenced by an external occurrence, such as abnormal movement of crude oil prices in Nigeria can be estimated using linear and non linear models since these models capture all of the distinct behaviours of the series.

The Markov witching model may is used. The MS-AR model is then one of the most appropriate techniques. [8]

Literature Review

According [6], applied seasonal ARIMA model of Nigerian monthly Crude Oil prices in US dollars. For the research, he used Eview software, and for detection and estimation, he used ARIMA. Time plot and autocorrelation or correlogram were used to show the seasonality. The consequence of the series' time plot indicates a high in 2008 and a digression in 2009. A high in 2008 and a deep trough in 2009 can still be observed using 12-month differencing. The time plot does not show seasonality. A SARIMA (0,1,1)*(1,1,1)₁₂ autocorrelation was discovered on the correlogram.

A research done by [15], Analyzed Intervention models of crude oil prices in Nigeria. The time plot of the series revealed an abrupt increase in the series and this called for intervention models. The knowledge was divided into three classes (actual series, pre-intervention and post-intervention series). The Augmented Dickey-Fuller (ADF) was used to test for unit root on each of the series, and they were all found to be non-stationary at different levels; however, they were non-stationary at the first difference (actual, pre, and post-intervention series). The pre-intervention model that reduced the Akaike Information Criterion (AIC) was the best of the eighteen models that were estimated.

Oil price volatility was modelled by [3] using macroeconomic variables in Nigeria. Different types of GARCH models were calculated with daily, weekly, and quarterly data in their paper. All of the macroeconomic variables studied (real GDP, interest rate, exchange rate, and oil price) are highly volatile; asymmetric models (TGARCH and EGARCH) outperform symmetric models (GARCH (1, 1) and GARCH – M), and oil price is a major source of macroeconomic instability in Nigeria. The Nigerian economy, as a result, is vulnerable to both internal and external shocks. As a result, they concluded that asymmetric models should be given more weight in dealing with Nigerian macroeconomic volatility, and oil price volatility should be considered an important variable in the study of Nigerian macroeconomic fluctuations.

The crude oil production in Nigeria was examined by [10], using the ARIMA model. The studies are important because the amount of crude oil produced determines the amount that will be refined and sold or exported as crude, which is the Nigerian economy's mainstay. Having a foreknowledge of the amount of crude oil that can be made, combined with appropriate budgeting, can aid the country's economic viability. The best model to match the crude oil output data was the multiplicative SARIMA (1, 1, 1) (0, 1, 1)₁₂ model. The forecast values from the fitted model agreed with the actual values, therefore, suggesting that the model could be used for forecasting the future quantity of crude oil that may be produced in the country

A Univariate linear ARIMA and GARCH was used by [11] in modelling and forecasting crude oil prices. The data used in this analysis consists of 189 monthly crude oil price observations in Nigeria. Autocorrelation function (ACF) and partial autocorrelation function (PACF) plots are used to determine the stationary state of the data set, which is then checked using the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) and Augmented Dickey-Fuller (ADF) test statistic. The price of crude oil has been discovered to be non-stationary. The same types of plots and statistics demonstrate that the data is stationary after taking the first difference of logarithmic values of the data sequence. Using parameters such as AIC, HQC, and SIC, the best ARIMA and GARCH models were chosen. The best model is one in which the values of the criteria are the smallest. As a result, the best models for forecasting crude oil price data series are ARIMA (3, 1, 1) and GARCH (2, 1).

Methodology

3.1 Source of Data

The researcher collected secondary data on crude oil price from statistical bulletins, Central Bank of Nigeria (CBN). The choice of data does not describe other variables ineffective, rather such variables appear to relate with each other in unique technique.

3.1.1 Time Plot

Time series data are non-stationary in nature. When presented with any time series data, the first step in the analysis is usually a time plot of data, to examine a simple trend movement of the series by time plot of the original series. The graph of crude oil prices was plotted against time to enable us to have an idea of the overall movement of the original data over the periods, whether the trend is constant or dies out with time. This plot also enables the researcher to know about the following: Trend (upward or downward) movement in the entire time, Seasonal fluctuation, Constant variances and the expansion and contraction period.[12]

3.2 Linear Models

3.2.1 Autoregressive (AR) Models of Order (p)

An autoregressive model is a time series model in which one uses the statistical properties of the past values of the series to predict future values. The general illustration of an autoregressive model of order p, AR(p) is [17]

$$X_t = \sum_{i=0}^p \sigma_i X_{t-i} + \tau_t \quad (3.1)$$

Generally, the Gaussian autoregressive of order p AR(p) process with mean π is given by

$$X_t - \pi = \sum_{i=0}^p \sigma_i (X_{t-i} - \pi) + \tau_t \quad (3.2)$$

Where the term ϵ_t is the error term and is called white noise, σ_0 , σ_1 and σ_p are unknown parameters, while X_t , X_{t-1} , X_{t-2} and X_{t-p} are estimated from the sample. Box and Jenkin (1976)

3.2.2 Stationarity Conditions for AR (P) Process

The characteristics polynomial of an AR (p) process, given as;

$$(1 - \sigma_1 B - \dots - \sigma_p B^p)X_t = \rho X_t = \tau_t \quad (3.3)$$

And the process must satisfy certain conditions for the process to be stationary. For instance, the first-order autoregressive:[4]

$$(1 - \beta_1 B)y_t = \tau_t \quad (3.4)$$

may be written as;

$$X_t = (1 - \sigma_1 B)^{-1} \tau_t = \sum_{j=0}^{\infty} \sigma_1^j X_{t-1} \quad (3.5)$$

Hence;

$$\psi(B) = (1 - \sigma_1 B)^{-1} = \sum_{j=0}^{\infty} \sigma_1^j B^j \quad (3.6)$$

This implies that the parameter σ_p , of an AR (p) process, must satisfy the condition $|\sigma_p| < 1$ to ensure stationarity.

In general, for an AR(p), the root of the characteristic equation $\rho(B) = 0$ must lie inside the unit circle to ensure stationarity.[14]

Moving Average (MA) Model

In terms of deviation from the series, a series X_t is said to follow a moving average process of order MA (q). mathematically represented in the form

$$X_t = \tau_t - \sum_{i=1}^q \sigma_i \tau_{t-i} \quad (3.7)$$

τ_t is the white noise and $\sigma_1, \sigma_2, \dots, \sigma_q$ are constants. This is known as the finite moving average of order q, ma (q). It should be noted that defining the MA polynomial with a negative or positive sign does change the properties of the model but only changes the algebraic signs of the MA coefficients.

3.8 Invertibility Condition for (MA) Model

For an $MA(q)$, the invertibility condition is that the roots of the characteristic polynomial lie inside the unit root circle. [4]

$$|1 - \sigma_1 B - \sigma_2 B^2 - \dots - \sigma_p B^p| < 1 \quad (3.8)$$

3.9 Autoregressive-Moving-Average Models (ARMA)

We have seen from above that the AR model includes lagged terms on the series itself and that the MA model includes lagged terms on the error term. By including both types of lagged terms, we arrive at the ARMA model. Therefore ARMA (p,q), where p is the order of autoregressive term and q the order of the moving-average term, can generally be represented as

$$X_t = \sum_{i=0}^p \sigma_i X_{t-i} + \tau_t - \sum_{i=1}^q \sigma_i \tau_{t-i} \quad (3.9)$$

A series $\{X_t\}$ is said to follow an autoregressive moving average model of orders p and q, designated ARMA (p, q), where σ_i and σ_i are constants such that the model is stationary as well as invertible and ε_t is a white noise process. Let model (3.9) be written specifically as;

$$A(B)X_t = B(L)\tau_t \quad (3.10)$$

$$\text{Where } A(B) = 1 - \sigma_1 B - \sigma_2 B^2 - \dots - \sigma_p B^p \quad (3.11)$$

$$B(L) = 1 + \sigma_1 B + \sigma_2 B^2 + \dots + \sigma_q B^q \quad (3.12)$$

B is the backshift operator defined by

$$B_t^k X_t = X_{t-k} \quad (3.13)$$

3.10 ARIMA Model with Differencing

Many series are non-stationary. For a non-stationary series $\{X_t\}$, Box – Jenkins proposed that differencing up to an appropriate order make it stationary. Suppose d is the minimum order of differencing necessary for stationarity to be attained. The dth difference of $\{X_t\}$ is denoted by $\{\Delta^d X_t\}$ where Δ^d is the difference operator defined by $\Delta^d = 1 - B$. If the series $\{\Delta^d X_t\}$ follows the model (3.12), then $\{X_t\}$ is said to follow an autoregressive integrated moving average model of order p, d and q designated as ARIMA (p, d, q).

The general ARIMA(p,d,q) model can be written as

$$\varphi(B)(1 - B)^d X_t = \theta(B)\varepsilon_t \quad (3.14)$$

3.3 Non-Linear Modelling

3.3.1 Markov Switching Model

Indefinite situations, the regime in operation at any point in time is directly observable. More generally if the regime is unobserved, the researcher must conduct inference about which models are allowed to switch from one state to another in each fixed number of regimes. A stochastic process assumed to have generated the regime shifts as part of the model, which allows variables to switch between regimes according to an unobserved Markov chain the process is represented by past values of the series. Regime switching models can be divided into two categories, threshold models and Markov-switching models. The primary difference between these approaches is how the state process is modelled. Threshold models, introduced by [9], assume that regime shifts are triggered by the level of observed variables about an unobserved threshold. Markov-switching models, introduced by [8], assume that the regime shifts according to a Markov chain.

Markov-switching models also assume that S_t is the unobserved state variable and y_t an observed variable. Contrast to threshold models, Markov-switching models Assume that, S_t follow a particular stochastic process, namely an N state Markov chain. The development of Markov chains is described by their transition probabilities, given by:

$$\Pr(S_t = i/S_{t-1} = j, S_{t-2} = q \dots \dots) = \Pr(S_t = i/S_{t-1} = j) = P_{ij} \quad (3.15)$$

Where conditional on a value of j , we assume $\sum_{i=1}^n Pr_{ij} = 1$. That is, the process specifies a complete probability distribution for S_t . In general, the Markov process allows regimes to

switch from one state to another. More than once restrictions can be placed on Pr_{ij} to restrict the order of regime shifts. [12]

3.2.2 Markov Switching Autoregressive Model (MS-AR)

The technique of using switching probability in non-linear models was first discussed by and a similar idea of modelling a non-linear series was also developed by [8] which emphasizes the aperiodic transition between the various state of the economic variable. The transition is driven by a hidden Markov state. A time series y_t follow an MS-AR model if it satisfies the following models.

$$X_t = \begin{cases} A_1 + \sum_{i=1}^p \sigma_{1,i} X_{t-i} + \tau_{1,t} & \text{if } s_t = 1 \\ \vdots \\ A_2 + \sum_{i=1}^p \sigma_{2,i} X_{t-i} + \tau_{2,t} & \text{if } s_t = 2 \end{cases} \quad (3.16)$$

In general, Markov switching autoregressive model of order p is represented by [2]

$$X_t = A_{s_t} + \sum_{i=1}^p \sigma_{s_t,i} X_{t-i} + \tau_{s_t,t} \quad \text{if } s_t = m \quad (3.17)$$

s_t follow a first-order Markov chain with transitional probability

$$\Pr(s_t = j/s_{t-1} = i) = w_1, \Pr(s_t = i/s_{t-1} = j) = w_2 \quad (3.18)$$

$\varepsilon_{s_t,t}$ is the error terms iid random variable with mean zero and infinite variances and independent of each other.[8] If p_{ij} is small, it's mean that the model tends to stay longer in a state i than in state j. The expected duration of the process is given by $\frac{1}{w_i}$ (the period the switching is to stay in state i), the Markov switching autoregressive models uses a hidden Markov chain to govern the transition probability from one conditional mean function to another.[9]

RESULT

In achieving the objectives of this study, monthly series on Crude oil prices (us dollar) in Nigeria from 2006 to 2019 (168 observations) used for the study. The raw data of crude oil prices were obtained from <https://www.centralbank.com>.

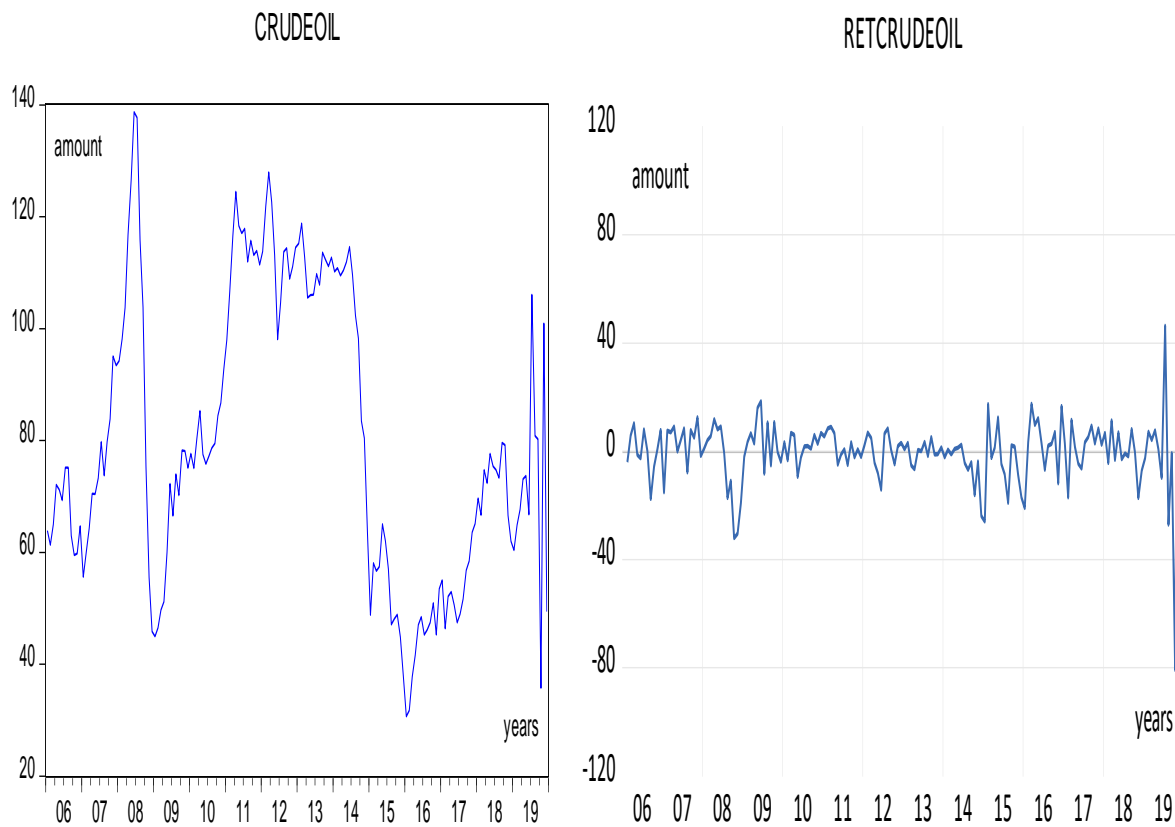


Figure 4.1 Time Plots of Crude Oil Prices at Levels and First differencing

A critical look at the time plot revealed upward and downward movement in the series, this means that the series exhibit a regime-switching pattern (a period of expansion and contraction in their movement), showing a period of 2 regimes in the variable of our studies. Therefore, a linear trend is present in the data. The presence of a trend in a series will make it not to be stationary (a series is said to be stationary if it has constant mean and variance). The time plot of first differences showing a stationary process, (meaning that the series has constant mean and variances)

Test for Stationarity

Table 4.2: Augmented Dickey-Fuller (ADF) Unit Roots Test

Variable	Levels		1 st Difference	
	Intercept	An Intercept, Linear Trend	Intercept	Intercept Linear Trend
Crude oil prices (cr_{edit})	-2.28331 (00.1786)	-2.510137 (0.3229)	-5.609171 (0.000)	-5.613260 (000)
Test critical values:				
1% level	-3.4731			
5% level	-2.8802			
10% level	-2.5768			

The variables of the study were tested for stationarity since the variables of the study cannot be applied for analysis unless it is established that the variables are stationary. Data on series was tested for stationarity to avoid the problem of spurious regression. The Augmented Dickey-Fuller (ADF) test was used to test for unit root on each of the variables, Table above shows the result of The Augmented Dickey-Fuller (ADF) test at the levels and first differences, constant, linear trend and probability values in brackets, at the level is greater than 0.05 (p-values >0.05) in all the variables, the result showed the presence of unit root since the series is non-stationary. Since the series is Non-stationary series and produces spurious regression, there is a need for the first difference of order. The p-value for all the variables at first differences was tested for stationarity and the series was found to be stationary. The probability values are (0.000) for the variables.[7]

Table 4.3: Estimation of Univariate Linear and Univariate Non-Linear Models.

Autoregressive Integrated Moving Model Of Crude Oil Prices				
S/N	Models	AIC	SC	LOG-LIKELIHOOD
1	ARIMA(111)	6.4506071	6.570305	-538.3845
2	ARIMA(110)	6.488184	6.544196	-538.7634
3	ARIMA(011)	6.514677	6.570688	-540.9755
4	ARIMA(112)	6.502152	6.595505	-537.9297
5	ARIMA(212)	6.495489	6.607512	-536.3733
6	ARIMA(012)	6.494051	6.565733	-538.2532
7	ARIMA(210)	6.493685	6.568378	-518.6912
Markov Switching Autoregressive Model Of Crude Oil Prices				
8	MS-AR(1)	6.345686	6.495661	-516.1288
9	MS-AR(2)	6.377319	6.565558	-516.1288

Seven models were estimated for univariate linear (ARIMA (p, d ,q)) and two for univariate non-linear (MS-AR(p)). the series was stationary at lag 1, the best model was selected base on minimise information criterion. MS-AR(1) was chosen for a series of crude oil prices with the following information criterion AIC (6.34586), SC (6.495061) and the largest log-likelihood (-516.1286). **The MS-AR model of Hamilton performs better than the linear autoregressive models propose by Box- Jenkins in examining the data of crude oil prices in Nigeria**

5.6 Markov Switching Autoregressive Model of Crude Oil Prices

The MS-AR(1) model of crude oil prices in both regime 1 & 2 can be represented mathematically as follow:

$$cr_{1,t} = 1.63244 - 0.337653cr_{1,t-1} \text{ if } s_t=1 \text{ (regime 1)} \quad (5.1)$$

$$cr_{2,t} = -2.018271 + 0.651515cr_{2,t-1} \text{ if } s_t=2 \text{ (regime 2)} \quad (5.2)$$

Where cr_t = represent crude oil prices at current values

$s_t = 1$ represent expansion and

$s_t = 2$ denote contraction in differences regime.

The Transition Probability from Regime 1 to Regime 2 is Presented below

$$P_r(1,2) = p_r(s(t)) = 2 / s(t - 1 = 1)$$

Variables	Transition Probability				Expected Duration			
	P_{11}	P_{12}	P_{21}	P_{22}	E_{11}	E_{12}	E_{21}	E_{22}
Crude Oil Prices	0.75215	0.247844	0.307290	.0.692710	4.034796	1.32951	1.4436	3.25425

5.6.1 The Transition Matrix Crude Oil Prices

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0.75215 & 0.247844 \\ 0.307290 & 0.692710 \end{bmatrix} \quad (5.3)$$

Where $P_{11} + P_{12} = 1$, $P_{21} + P_{22} = 1$

P_{11} Denote the probability of transitioning to expansion in the next period given that the current state is in expansion. P_{12} Denote the probability of transitioning to a contraction in the next period given that the current state is in expansion. P_{21} Denote the probability of transitioning to expansion in the next period given that the current state is in contraction. P_{22} Denote the probability of transitioning to a contraction in the next period given that the current state is in contraction.

5.6.2 Expected Duration

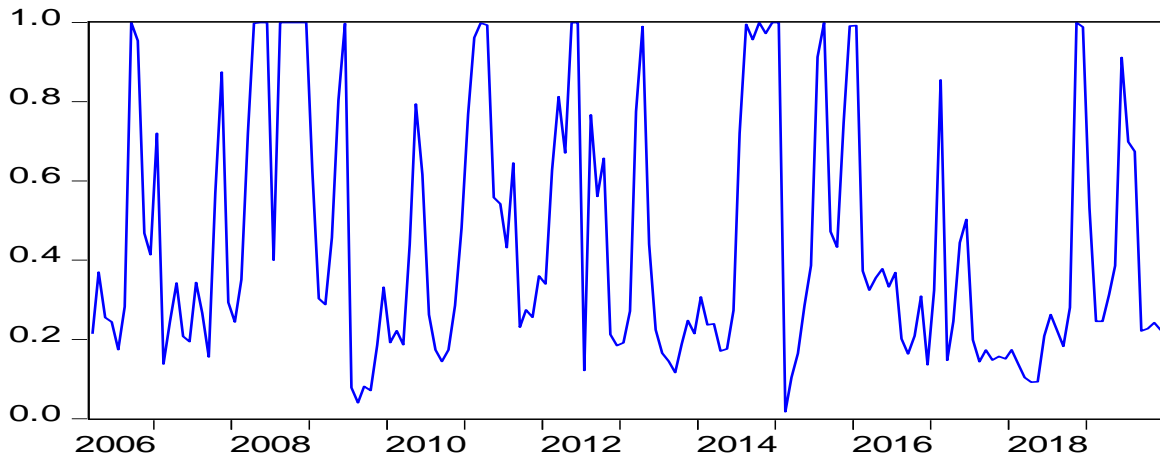
The expected time spent in each state is called the expected duration. If D_1 is the expected duration spend in state 1, these can be denoted as

$$E(D_1) = \frac{1}{1 - p_{11}}$$

The closer P_{11} is to 1 the higher is the expected duration of state 1. Detained in table 4.4.

Markov Switching Smoothed Regime Probabilities

$P(S(t)= 1)$



$P(S(t)= 2)$

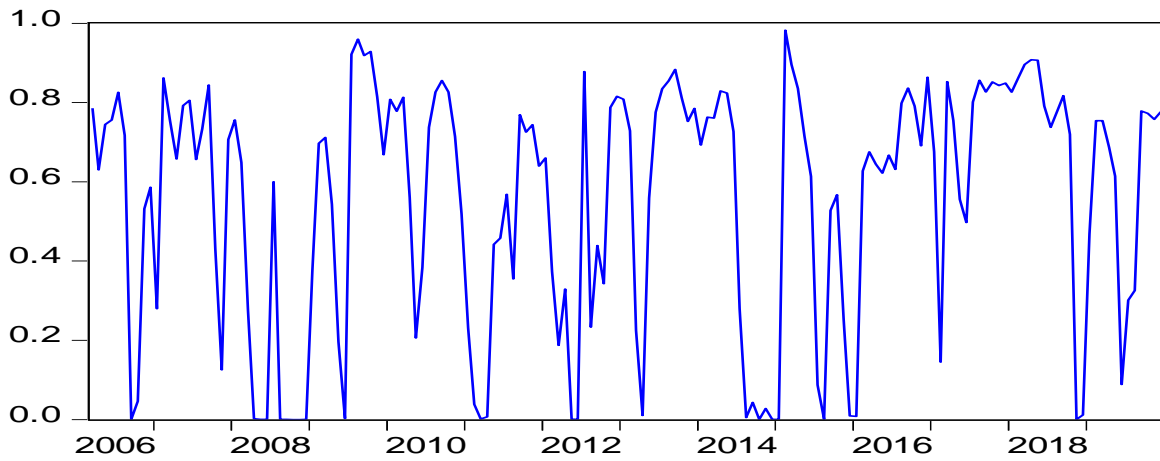
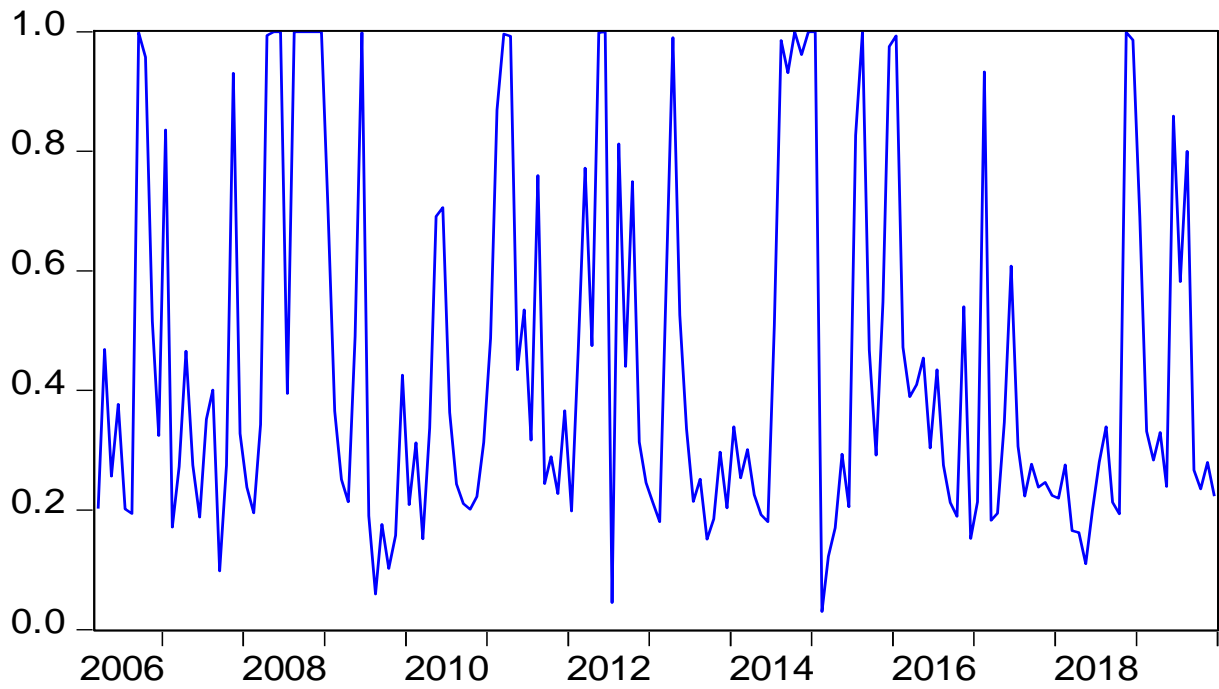


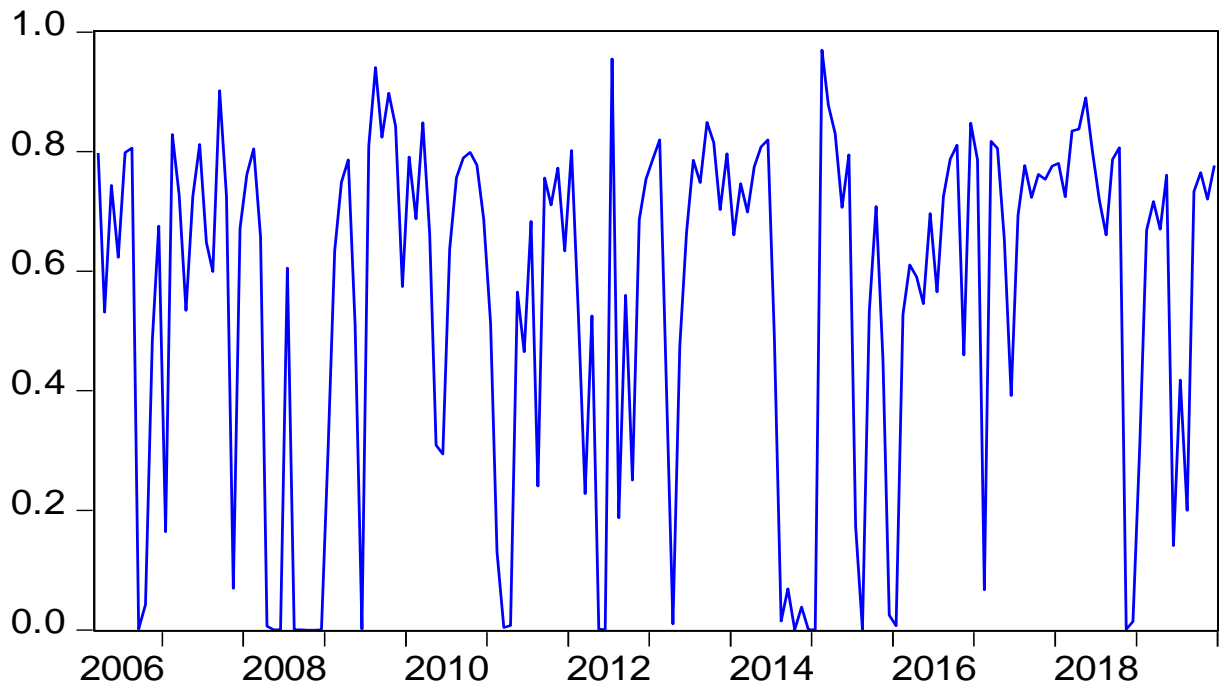
Figure 4.6: Smoothed Probabilities Crude Oil Prices in both Regime 1 & 2

Markov Switching Filtered Regime Probabilities

$$P(S(t)= 1)$$



$$P(S(t)= 2)$$



Filtered Probabilities of Crude Oil Prices Regime 1 & 2

5.6.3 Smoothed and Filtered Probability of Crude Oil Prices

The smoothed probabilities are shown above which provide inference on s_t conditional on all available samples information, for example, exogenous and endogenous switching process. In regime one $pr(s_t = 1)$, the expansion we find out that the smoothing process fluctuates up and down. In regime two $pr(s_t = 2)$, contraction smoothing is similar to regime one but in the opposite direction, this is an indication of unstable prices of crude oil in the international market which may lead to the financial crisis.

Filtered Probability of crude oil prices showed in the above figure, This is the probabilities that the unobserved state for a switching model is in a particular regime in time t is the condition on observing sample information up to time t. the filtered probability have a similar pattern with the smoothed probability.

Table 4.5: Forecast Value of Crude Oil Prices from January 2020 to December 2020

Month	Forecast Value for 2020 January -2020 December for Crude Oil Prices		
	Regime1	Regime2	Average
January	65.664	62.8029	64.23345
February	66.835	59.809364	63.32215
March	68.071	55.84	61.9285
April	69.287	51.236	60.2615
May	70.509	46.219	58.364
June	71.728	40.9316	56.3298
July	72.949	35.4685	54.20875
August	74.1696	29.890951	52.03
September	75.39	24.238851	49.814455
October	76.61	18.53885	47.54425
November	77.83	12.80681	45.3184
December	79.05	7.053851	43.05

5.6.4 Forecast Equation of Crude Oil Prices for 2020 January -2020 December at

Regime 1 Using MS-AR(1) model

$$cr_t = 1.63244 - 0.337653cr_{t-1} \text{ for January 2020} \quad (5.4)$$

$$cr_{t+1} = 1.63244 - 0.337653cr_t \text{ for February 2020}$$

(5.5)

$$cr_{t+11} = 1.63244 - 0.337653cr_{t+10} \text{ for December 2020} \quad (5.6)$$

5.6.5 Forecast Equation of Crude Oil Prices for January to December 2020 at Regime 2

$$cr_t = -2.018271 + 0.651515cr_{t-1} \text{ for January 2020} \quad (5.7)$$

$$cr_{t+1} = -2.018271 + 0.651515cr_t \text{ for February 2020} \quad 5.8$$

$$cr_{t+11} = -2.018271 + 0.651515cr_{t+10} \text{ for December 2020} \quad (5.9)$$

Conclusion

This paper compared the autoregressive integrated moving average (ARIMA) model proposed by Box and Jenkins and Markov switching autoregressive model (MS-AR) proposed by Hamilton in modelling crude oil prices in Nigeria. The data was obtained from CBN statistical Bulletin. The time plot of the series indicates two "separate" regime (expansion and contraction), the Augmented Dickey-Fuller, unit root test was used to test for stationarity of the variable and the series was stationary at first differences. Seven models were estimated for univariate linear univariate and two models for non-linear models. The best model was selected based on minimise information criterion. MS(2)-AR(1) was chosen for crude oil prices. These models were used to forecast the values of the series for one year ahead.

Competing Interests

Authors have declared that no competing interests exist.

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