EVALUATING MEASURE OF MODIFIED ROTATABILITY FOR SECOND DEGREE POLYNOMIAL DESIGN USING BALANCED INCOMPLETE BLOCK DESIGNS

Abstract

Box and Hunter (1957) introduced the concept of rotatability. It is an important design criterion for response surface methodology (RSM). In this paper, evaluating measure of modified rotatability for second degree polynomial design using balanced incomplete block designs ($3 \leq v \leq 11$: v-number of factors) which enables us to assess the degree of modified rotatability for a given response surface designs at different values of rotatability is recommended.

Keywords: Response surface method, modified rotatable designs, incomplete block designs.

1. Introduction

Response surface process is a collection of mathematical and statistical techniques appropriate for analysing problems in which several independent variables influence a dependent variable. The regressor variables are often called input or explanatory variables and the regressand variable is often the response variable. An important development of response surface designs was the introduction of rotatable designs suggested by Box and Hunter (1957). Rotatable designs using balanced incomplete block designs (BIBD) was proposed by Das and Narasimham (1962). A design is said to be rotatable, if the variance of the response estimate is a function only of the distance of the point from the design centre. Das et al. (1999) offered modified second order response surface designs. Park et al. (1993) developed measure of rotatability for second degree polynomial designs.

A lot of research work carried out by Victorbabu and some authors in the area of modified rotatability, measure of rotatability (Victorbabu and Vasundharadevi (2005), Victorbabu et al. (2006), Victorbabu (2008), Victorbabu et al. (2008), Victorbabu and Vasundharadevi (2008, 2009), Victorbabu and Surekha (2012), Victorbabu and Surekha (2013), Victorbabu and Chiranjeevi (2018), Chiranjeevi and Victorbabu (2020), Victorbabu and Surekha (2015), Victorbabu and Jyostna (2017)).

Some work contributed on tri-diagonal, intra-class correlated structure of errors on second order rotatable designs (SORD) by Rajyalakshmi and Victorbabu (2014, 2015, 2016). Specifically Jyostna and Victorbabu (2021) studied evaluating measure of modified rotatability for second degree polynomial designs using central composite designs. These measures are useful to enable us to assess the degree of modified rotatability for a given second degree polynomial designs. In this work "evaluating measure of modified rotatability for second degree polynomial design using balanced incomplete block designs ($3 \leq v \leq 11$: v-number of factors) which enables us to assess the degree of modified rotatability for a given response surface designs at different values of rotatability is recommended".

2. SORD - Conditions

Suppose we want to use the second degree polynomial model D=$((x_{ru}))$ to fit the surface,

$$Y_q = b_0 + \sum_{r=1}^{v} b_r x_{rq} + \sum_{r=1}^{v} b_{rr} x_{rq}^2 + \sum\sum_{r<s} b_{rs} x_{rq} x_{sq} + e_q \quad (1)$$

where x_{rq} denotes the level of the r^{th} factor (r =1,2,…,v) in the q^{th} run (q=1,2,…,N) of the experiment, e_q's are uncorrelated random errors with mean zero and variance σ^2 is said to be rotatable design of second order, if the variance of the estimated response of $\hat{Y}_q$ from the fitted surface is only a function of the distance ($d^2 = \sum_{r=1}^{v} x_r^2$) of the point ($x_1, x_2, \ldots, x_q$) from the origin (centre) of the design. Such a spherical variance function for estimation of second degree polynomial is achieved if the design points satisfy the following conditions [cf. Box and Hunter (1957), Das and Narasimham (1962)].

1. $\sum x_{rq} = 0$, $\sum x_{rq} x_{sq} = 0$, $\sum x_{rq} x_{sq}^2 = 0$, $\sum x_{rq} x_{sq} x_{tq} = 0$, $\sum x_{rq}^3 = 0$, $\sum x_{rq} x_{sq}^3 = 0$, $\sum x_{rq} x_{sq} x_{tq}^2 = 0$, $\sum x_{rq} x_{sq} x_{tq} x_{uq} = 0$; for $r \neq s \neq t \neq u$; (2)
2. (i) $\sum x_{rq}^2 =$ constant = $N\gamma_2$;

   (ii) $\sum x_{rq}^4 =$ constant = $cN\gamma_4$; for all r (3)
3. $\sum x_{rq}^2 x_{sq}^2 =$ constant = $N\gamma_4$; for $r \neq s$ (4)
4. $\sum x_{rq}^4 = c \sum x_{rq}^2 x_{sq}^2$ (5)
5. $\dfrac{\gamma_4}{\gamma_2^2} > \dfrac{v}{(c+v-1)}$ (6)

where c, γ_2 and γ_4 are constants and the summation is over the design points.

The variances and covariances of the estimated parameters become,

$$V(\hat{b}_0) = \frac{\gamma_4(c+v-1)\sigma^2}{N\left[\gamma_4(c+v-1)-v\gamma_2^2\right]},$$

$$V(\hat{b}_r) = \frac{\sigma^2}{N\gamma_2},$$

$$V(\hat{b}_{rs}) = \frac{\sigma^2}{N\gamma_4},$$

$$V(\hat{b}_{rr}) = \frac{\sigma^2}{(c-1)N\gamma_4}\left[\frac{\gamma_4(c+v-2)-(v-1)\gamma_2^2}{\gamma_4(c+v-1)-v\gamma_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{rr}) = \frac{-\gamma_2\sigma^2}{N[\gamma_4(c+v-1)-v\gamma_2^2]},$$

$$Cov(\hat{b}_{rr},\hat{b}_{ss}) = \frac{(\gamma_2^2-\gamma_4)\sigma^2}{(c-1)N\lambda_4[\gamma_4(c+v-1)-v\gamma_2^2]} \tag{7}$$

and other covariances are zero.

3. Modified SORD - Conditions

The most widely used design for fitting a second degree polynomial is the central composite design (CCD). Central composite designs are constructed by adding suitable factorial combinations to those obtained from $\frac{1}{2^p}\times 2^v$ fractional factorial design (here $2^{t(v)} = \frac{1}{2^p}\times 2^v$ denotes a suitable fractional replicate of 2^v, in which no interaction with less than five factors is confounded). In coded form the points of $2^v(2^{t(v)})$ factorial have coordinates $(\pm\alpha,\pm\alpha,...,\pm\alpha)$ and $2v$ axial points have coordinates of the form ($(\pm\beta, 0,...0), (0, \pm\beta,...0), . . . ,(0, 0, ..., \pm\beta)$) etc., and n_0 central points. The usual method of construction of rotatable designs using CCD is to take combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant. All such combinations form a design. Generally, rotatable designs of second order need at least five levels (suitably coded) at $0, \pm\alpha, \pm\beta$ for all factors ($(0, 0, \ldots 0)$)- chosen centre of the design, unknown level 'α' and 'β' are to be chosen suitably to satisfy the conditions of the rotatability) generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively, by putting some restrictions indicating some relation among $\sum x_{rq}^2, \sum x_{rq}^4$ and $\sum x_{rq}^2 x_{sq}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In rotatable designs of second order the restriction used is $\sum x_{rq}^4 = 3\sum x_{rq}^2 x_{sq}^2$, i.e., $c=3$. Das et al (1999) proposed the restriction $(\sum x_{rq}^2)^2 = N\sum x_{rq}^2 x_{sq}^2$ i.e., $\gamma_2^2 = \gamma_4$ to get another series of symmetrical second order response surface designs, which will provide more precise estimates of response at specific points of interest than what is available from the corresponding existing designs. On simplification of (7) using the above condition $(\sum x_{rq}^2)^2 = N\sum x_{rq}^2 x_{sq}^2$, the variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}$$

$$V(\hat{b}_r) = \frac{\sigma^2}{N\sqrt{\gamma_4}}$$

$$V(\hat{b}_{rs}) = \frac{\sigma^2}{N\gamma_4}$$

$$V(\hat{b}_{rr}) = \frac{\sigma^2}{(c-1)N\gamma_4}$$

$$Cov(\hat{b}_0, \hat{b}_{rr}) = \frac{-\sigma^2}{N\sqrt{\gamma_4}(c-1)} \qquad (8)$$

and other covariances are zero. Above modifications of the variances and covariances affect the variance of the estimated response at specific points considerably. Applying these variances and covariances, $V(\hat{Y}_q)$ at any point can be obtained. Let $\hat{y}_q$ denote the estimated response at the point $(x_{1q}, x_{2q}, \ldots x_{vq})$. Then,

$$V(\hat{Y}_q) = V(\hat{b}_0) + d^2[V(\hat{b}_r) + 2cov(\hat{b}_0, \hat{b}_{rr})] + d^4 V(\hat{b}_{rr}) + (\sum x_{rq}^2 x_{sq}^2)[(c-3)\sigma^2 / (c-1)N\gamma_4]$$

The study of modified response surface designs is the same as for SORD except that instead of taking $c=3$ the new restriction $(\sum x_{rq}^2)^2 = N\sum x_{rq}^2 x_{sq}^2$ is to be used and this condition will provide different unknown values are involved. (cf. Das et al. 1999).

4. Conditions for Evaluating Measure of Rotatability for Second Degree Polynomial Designs

Following Box and Hunter (1957), Das and Narasimham (1962), Park et al (1993), conditions (2) to (6) and (7) provide the necessary and sufficient conditions for evaluating rotatability for any general second degree polynomial designs. Further we have,

$V(b_r)$ are equal for r,

$V(b_{rr})$ are equal for r,

$V(b_{rs})$ are equal for r, s, where $r \neq s$,

$$Cov(b_r, b_{rr}) = Cov(b_r, b_{rs}) = Cov(b_{rr}, b_{rs}) = Cov(b_{rs}, b_{ru}) = 0 \text{ for all } r \neq s, s \neq u, u \neq r. \quad (9)$$

Park et al. (1993) suggested that if the conditions in (2) to (6) together with (7) and (9) are satisfied, then the following measure $(J_v(D))$ can be used to assess the degree of rotatability for any general second degree polynomial design (cf. Park et al., 1993).

$$J_v(D) = \frac{1}{1+R_v(D)}, \quad (10)$$

here

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \frac{6v\left[V(\hat{b}_{rs}) + 2\,cov(\hat{b}_{rr}, \hat{b}_{ss}) - 2V(\hat{b}_{rr})\right]^2 (v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \quad (11)$$

and the scaling factor g.

On simplification, numerator of (11), $[V(\hat{b}_{rs}) + 2\,cov(\hat{b}_{rr}, \hat{b}_{ss}) - 2V(\hat{b}_{rr})]$ using (7) reduces to $(c\text{-}3)\sigma^2 / (c\text{-}1)N\lambda_4$. Thus $R_v(D)$ reduces to

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v[(c-3)\sigma^2]^2(v-1)}{[(c-1)N\gamma_4]^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right) \qquad (12)$$

Note: For SORD, we take $c = 3$. Substituting the value of $'c'$ and on simplification of (12) we get $R_v(D)$ is zero. Hence from (10), we get $J_v(D)$ is one if and only if a design is rotatable and less than one then it is nearly rotatable design.

5. Modified Rotatability for Second Degree Polynomial Designs using BIBD

Balanced Incomplete Block Design (BIBD): A BIBD is denoted with the parameters (v,b,r,k,λ). An arrangement of v treatments in b blocks each containing $k(< v)$ treatments, if (i) every treatment occurs at most once in each block, (ii) every treatment occurs in exactly r blocks and (iii) every pair of treatments occurs together in λ blocks.

The result of modified rotatability for second degree polynomial designs using BIBD is suggested here (cf. Victorbabu and Vasundharadevi 2005). Let (v,b,r,k,λ) represent a BIBD, where $2^{t(k)}$ denotes a fractional replicate of 2^k with ± 1 levels in which no interaction with less than five factors is confounded. Let $[1\text{-}(v,b,r,k,\lambda)]$ represent the design points generated from the transpose of the incidence matrix given BIBD. Let $[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}$ give $b2^{t(k)}$ design points generated from BIBD by "multiplication" (cf. Raghavarao, 1971). Repeat the set of $b2^{t(k)}$ design points z_1 times. Let $(\pm\alpha,0,0,\ldots 0)2^1$ represent the design points generated from $(\pm\alpha,0,0,\ldots 0)$ point set when $r < 3\lambda$ repeat this set of additional design points, say z_2 times. Let n_0 be the number of central points in modified SORD and U represent combination of the design points generated from different sets of points.

The design points, $z_1[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}\ U\ z_2(\pm\alpha,0,0,\ldots.0)2^1\ U\ (n_0)$ will give a v-dimensional modified SORD in $N = \dfrac{(z_1 r 2^{t(k)} + z_2 2\alpha^2)^2}{z_1\lambda 2^{t(k)}}$ design points if,

$$\alpha^4 = \frac{z_1(3\lambda - r)2^{t(k)-1}}{z_2},$$

$$n_0 = \frac{(z_1 r 2^{t(k)} + z_2 2\alpha^2)^2}{z_1 \lambda 2^{t(k)}} - [z_1 b 2^{t(k)} + z_2 2v]$$ and n_0 turns out to be an integer.

6. Evaluating measure of Rotatability for Second Degree Polynomial Designs using BIBD

Here we suggest the result of evaluating measure of rotatability for second degree polynomial designs using BIBD. (cf. Victorbabu and Surekha, 2015). Let (v,b,r,k,λ) represent a BIBD. The design points, $z_1[1-(v,b,r,k,\lambda)]2^{t(k)}$ U $z_2(\alpha,0,0,....0)2^1$ U (n_0) will give a evaluating measure of rotatability for second degree polynomial designs using BIBD in $N = z_1 b 2^{t(k)} + z_2 2v + n_0$ design points with level 'α' prefixed and $c = \frac{r2^{t(k)} z_1 + z_2 2\alpha^4}{\lambda 2^{t(k)} z_1}$ (for $z_1 = 1, z_2 = 1$).

Evaluating measure of rotatability values for second degree polynomial using BIBD is given below. We have

$$R_v(D) = \left[\frac{(c-3)}{(c-1)}\right]^2 \frac{6v(v-1)}{\gamma_4^2 (v+2)^2 (v+4)(v+6)(v+8) g^8}$$

here

$$g = \begin{cases} \dfrac{1}{\alpha}, & \text{if } \alpha < \sqrt{\dfrac{z_1 (b-r) 2^{t(k)-1}}{z_2} + v} \\[2ex] \dfrac{1}{\sqrt{\dfrac{z_1 (b-r) 2^{t(k)-1}}{z_2} + v}}, & \text{if } \alpha > \sqrt{\dfrac{z_1 (b-r) 2^{t(k)-1}}{z_2} + v} \end{cases}$$

$$J_v(D) = \frac{1}{1 + R_v(D)}$$

If $J_v(D)$ is one if and only if the design is rotatable, and it is less than one for a non-rotatable designs.

7. Evaluating Measure of Modified Rotatability for Second Degree Polynomial Designs using BIBD

The proposed method for evaluating measure of modified rotatability for second order response surface designs using BIBD when $r < 3\lambda$ is suggested here. Let (v,b,r,k,λ) represent a BIBD. $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels, such that no interaction with less than five factors is confounded. Let $[1\text{-}(v,b,r,k,\lambda)]$ represent the design points generated from the transpose of incidence matrix BIBD, Let $[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from BIBD by "multiplication". Repeat these $b2^{t(k)}$ design points z_1 times. Let $(\pm\alpha,0,...,0)2^1$ denote the design points generated from $(\pm\alpha,0,...,0)$ point set. Repeat this set of additional design points say z_2 times and n_0 be the number of central points. The method of evaluating measure of modified rotatability for second degree polynomial designs using BIBD is suggested as follows.

The design points,

$z_1[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}\ \mathrm{U}\ z_2(\pm\alpha,0,0,....0)2^1\ \mathrm{U}\ (n_0)$ generated from BIBD, we have,

$$\sum x_{rq}^2 = z_1 r 2^{t(k)} + z_2 2\alpha^2 = N\gamma_2 \qquad (13)$$

$$\sum x_{rq}^4 = z_1 r 2^{t(k)} + z_2 2\alpha^4 = cN\gamma_4 \qquad (14)$$

$$\sum x_{rq}^2 x_{sq}^2 = z_1 \lambda 2^{t(k)} = N\gamma_4 \qquad (15)$$

To make the design rotatable, we take $c = 3$. From equations (14) and (15), we have

$$\alpha^4 = \frac{z_1(3\lambda - r)2^{t(k)-1}}{z_2},$$

The modified condition $(\sum x_{rq}^2)^2 = N\sum x_{rq}^2 x_{sq}^2$ leads to N which is given by

$N = \dfrac{(z_1 r 2^{t(k)} + 2z_2\alpha^2)^2}{z_1\lambda 2^{t(k)}}$ alternatively N may be obtained directly as $z_1 b 2^{t(k)} + z_2 2v + n_0$, where

n_0 is given by $n_0 = \dfrac{(z_1 r 2^{t(k)} + z_2 2\alpha^2)^2}{z_1\lambda 2^{t(k)}} - [z_1 b 2^{t(k)} + z_2 2v]$ and n_0 turns out to be an integer. From equations (13) and (15) and on simplification we get $\lambda_2 = \dfrac{z_1 r 2^{t(k)} + z_2 2\alpha^2}{N}$ and $\lambda_4 = \dfrac{z_1\lambda 2^{t(k)}}{N}$.

To obtain evaluating measure of modified rotatability for second degree polynomial designs using BIBD, we have

$$J_v(D) = \frac{1}{1+R_v(D)}$$

$$R_v(D) = \left[\frac{(c-3)}{(c-1)}\right]^2 \frac{6v(v-1)}{\gamma_4^2(v+2)^2(v+4)(v+6)(v+8)g^8},$$

here g is a scaling factor,

$$g = \begin{cases} \frac{1}{\alpha}, \quad \text{if} \quad \alpha < \sqrt{\frac{z_1(b-r)2^{t(k)-1}}{z_2} + v} \\ \frac{1}{\sqrt{\frac{z_1(b-r)2^{t(k)-1}}{z_2} + v}} \quad \text{otherwise} \end{cases}$$

The following table gives the values of a evaluating measure of modified rotatability for second degree polynomial designs using BIBD. It can be verified that $J_v(D)$ is 1 if and only if the design is modified rotatable, and it is smaller than one for nearly modified rotatable designs.

Example: We illustrate the evaluating measure of modified rotatability for second degree polynomial for $v=5$ factors with the help of a BIBD $(v=5,b=10,r=6,k=3,\lambda=3)$. The design points,

$z_1[1-(v=5,b=10,r=6,k=3,\lambda=3)]2^3 \cup z_2(\pm\alpha,0,0,....0)2^1 \cup (n_0)$ will give a measure of modified rotatability for second degree polynomial in N=150 design points. From (13), (14) and (15), we have

$$\sum x_{rq}^2 = z_1 48 + z_2 2\alpha^2 = N\gamma_2 \tag{16}$$

$$\sum x_{rq}^4 = z_1 48 + z_2 2\alpha^4 = cN\lambda_4 \tag{17}$$

$$\sum x_{rq}^2 x_{sq}^2 = z_1 24 = N\gamma_4 \tag{18}$$

From equations (17) and (18) with rotatability value $c=3$, $z_1=1$ and $z_2=3$, we get $\alpha^4 = 4 \Rightarrow \alpha^2 = 2 \Rightarrow \alpha=1.414214$. From equations (16) and (18) using the modified condition with $(\gamma_2^2=\gamma_4)$ along with $\alpha^2=2$, $z_1=1$ and $z_2=3$, we get $N=150$, $n_0=40$. For modified SORD we get $J_v(D)=1$ by taking $\alpha=1.414214$ and scaling factor $g=0.7071$. Then the design is modified SORD using BIBD.

Instead of taking $\alpha=1.414214$ if we take $\alpha=2.2$ for the above BIBD $(v=5,b=10,r=6,k=3,\lambda=3)$ from equations (17) and (18), we get $c=7.8564$. The scaling factor $g=0.5578$, $R_v(D)=2.2199$ and $J_v(D)=0.3106$. Here $J_v(D)$ becomes smaller it deviates from modified rotatability.

Conclusion: The evaluating measure of modified rotatability for second degree polynomial designs using BIBD, at different values of $'\alpha'$ for $3 \le v \le 11$. It can be verified that $J_v(D)$ is one if and only if the design is modified rotatable design and it is less than one for a nearly modified rotatable design.

Table1. Evaluating measure of modified rotatability for second degree polynomial using BIBD

(3,3,2,2,1), $N=50$, $z_1=2, z_2=1$, $n_0=20$, $\alpha=1.414214$				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.25	1	0.5865	0.6303
1.3	2.7140	0.7692	0.3699	0.7299
*1.414214	3	0.7071	0	1
1.6	3.6384	0.625	4.0965	0.1962
1.9	5.2580	0.6148	22.4483	0.0426
2.2	7.8564	0.6148	40.0477	0.0244
2.5	11.7656	0.6148	52.9211	0.0185
2.8	17.3664	0.6148	61.5079	0.0159
3.1	25.0880	0.6148	67.1201	0.0147

(4,4,3,3,2), $N=81$, $z_1=1, z_2=3$, $n_0=25$, $\alpha=1.414214$				
α	c	g	$R_v(D)$	$J_v(D)$
1	1.875	1	0.16875	0.8556
1.3	2.5710	0.7692	0.0621	0.9415
*1.414214	3	0.7071	0	1
1.6	3.9576	0.6580	0.3044	0.7666
1.9	6.3870	0.6580	1.1479	0.4656
2.2	10.2846	0.6580	1.7875	0.3587

2.5	16.1484	0.6580	2.1876	0.3137
2.8	24.5496	0.6580	2.4314	0.2914
3.1	36.1320	0.6580	2.5825	0.2791

(5,10,6,3,3), N= 150, z_1 =1, z_2 = 3, n_0 = 40, α = 1.414214				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.25	1	0.0149	0.9854
1.3	2.7140	0.7692	0.0094	0.9907
*1.414214	3	0.7071	0	1
1.6	3.6384	0.625	0.1042	0.9056
1.9	5.2580	0.5578	1.2443	0.4456
2.2	7.8564	0.5578	2.2199	0.3106
2.5	11.7656	0.5578	2.9335	0.2542
2.8	17.3664	0.5578	3.4095	0.2268
3.1	25.0880	0.5578	3.7206	0.2118

(6,10,5,3,2), N= 121, z_1 =1, z_2 = 1, n_0 = 29, α = 1.414214				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.625	1	0.0044	0.9957
1.3	2.8570	0.7692	0.004	0.9960
*1.414214	3	0.7071	0	1
1.6	3.3192	0.625	0.0667	0.9374
1.9	4.1290	0.5263	1.8138	0.3554
2.2	5.4282	0.4545	13.5355	0.0688
2.5	7.3828	0.4428	26.1461	0.0368
2.8	10.1832	0.4429	33.9292	0.0286
3.1	14.0440	0.4429	39.7519	0.0245

(7,7,4,4,2), N= 162, z_1 =1, z_2 = 1, n_0 = 36, α = 2				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.0625	1	0.0138	0.9864
1.3	2.1785	0.7692	0.0704	0.9342
1.6	2.4096	0.625	0.1339	0.8819
1.9	2.8145	0.5263	0.0315	0.9694
*2	3	0.5	0	1
2.2	3.4641	0.4545	0.3459	0.7430
2.5	4.4414	0.4	4.7560	0.1737
2.8	5.8416	0.3571	23.1226	0.0415
3.1	7.7720	0.3226	75.2463	0.0131

(8,14,7,4,3), N= 300, z_1 =1, z_2 = 1, n_0 = 60, α = 2				
α	c	g	$R_v(D)$	$J_v(D)$

1	2.375	1	0.0014	0.9986
1.3	2.4523	0.7692	0.0078	0.9922
1.6	2.6064	0.625	0.0175	0.9828
1.9	2.8763	0.5263	0.005	0.9950
*2	3	0.5	0	1
2.2	3.3094	0.4545	0.06703	0.9372
2.5	3.9609	0.4	1.0937	0.4776
2.8	4.8944	0.3571	6.084	0.1411
3.1	6.1813	0.3535	10.5089	0.0869

(9,18,8,4,3), N= 726, z_1 = 2, z_2 = 1, n_0 = 132, α = 2				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.6875	1	0.0021	0.9998
1.3	2.7262	0.7692	0.0017	0.9988
1.6	2.8032	0.625	0.0315	0.9695
1.9	2.9382	0.5263	0.001	0.8480
*2	3	0.5	0	1
2.2	3.1547	0.4545	0.0166	0.9837
2.5	3.4805	0.4	0.3357	0.7487
2.8	3.9472	0.3571	2.2882	0.3041
3.1	4.5907	0.3226	9.8146	0.0925

(10,18,9,5,4), N= 441, z_1 =1, z_2 = 6, n_0 = 33, α = 1.414214				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.4375	1	0.0004	0.9996
1.3	2.7855	0.7692	0.0003	0.9997
*1.414214	3	0.7071	0	1
1.6	3.4788	0.625	0.0045	0.9955
1.9	4.6935	0.5263	0.1017	0.9077
2.2	6.6423	0.4671	0.5745	0.6351
2.5	9.5742	0.4617	0.8105	0.5523
2.8	13.7748	0.4617	0.9807	0.5048
3.1	19.5660	0.4617	1.0975	0.4767

(11,11,5,5,2), N= 242, z_1 =1, z_2 = 2, n_0 = 22, α = 1.414214				
α	c	g	$R_v(D)$	$J_v(D)$
1	2.625	1	0.0025	0.9976
1.3	2.8570	0.7692	0.0022	0.9978
1.414214	3	0.7071	0	1
1.6	3.3192	0.625	0.0375	0.9638
1.9	4.1290	0.5263	1.0193	0.4952
2.2	5.4282	0.4545	7.6066	0.1162
2.5	7.3828	0.4111	26.6264	0.0362

2.8	10.1832	0.4111	34.5524	0.02812
3.1	14.044	0.4111	40.482	0.02411

*indicates modified rotatability value using BIBD. (cf. Victorbabu and Vasundharadevi (2005))

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