Original Research Article

RADIATIVE EFFECTS ON UNSTEADY MHD COUETTE FLOW THROUGH A PARALLEL PLATE WITH CONSTANT PRESSURE GRADIENT

ABSTRACT

In this paper, the unsteady MHD Couette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates under the influence of thermal radiation and chemical reaction is investigated. A uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to the constant pressure gradient. The transformed conservation equations are solved analytically subject to physically appropriate boundary conditions by using theEigenfunction expansion technique. The influence of (some)a number of emerging non-dimensional parameters namely, pressure gradient, suction parameter, radiation parameter, and Hartman number are examined in detail. It is observed that the primary velocity is increased with increasing pressure gradient while theincrease in radiation parameter leads to adecrease in the thermal profile of the flow.

Keywords: Eigenfunction expansion,Magnetohydrodynamics(MHD), constant pressure gradient, suction, Hall current.

1. Introduction

The dynamics of fluids through theporous channel (have)has been a popular area of research regarding to numerous increasing applications in chemical, mechanical, and material process engineering. Examples of such fluid includes(include) clay coating, coal, oil slurries, shampoo, paints cosmetic products, grease, custard and physiological liquids (blood, bile, and synovial fluid). Over the years, considerable interest has been observed on the effect of MHD in viscous, incompressible, non-Newtonian fluid flow with heat transfer. These interests on non-Newtonian fluids are owed to its important applications in various branches of science, engineering and technology, particularly in chemical and nuclear industries, material processing, geophysics and bio-engineering. In view of these applications, an extensive range of mathematical models have been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. However, different non-Newtonian fluid models have been presented by researchers and solved using various types of analytical and computational schemes.

Sayed-Ahmed *et al.*[1] investigated time dependent pressure gradient effect on unsteady MHDCouette flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non-conducting porous plates with heat transfer under anexponential decaying pressure gradient.Olayiwola[2] investigated the modeling and simulation of combustion fronts in porous media.Jana *et al.* [3] examinedCouette flow through a porous medium in a rotating system. In another related work, Seth *et al.*[4] studied the effects of rotation and magnetic field on unsteady Couette flow in a porous channel. Seth *et al.*[5] studied the unsteady hydromagneticCouette flow within porous plates in a rotating system. Recently, Sharma &Yadav[6] considered Heat transfer through three dimensional Couette flow between a stationary porous plate bounded by porous medium and moving porous plates. Sharma *et al.*[7] investigated the steady laminar flow and heat transfer of a non-Newtonian fluid through a straight horizontal porous channel in the presence of aheat source.Olayiwola&Ayeni[8] examined a mathematical model and simulation of In-situ combustion in porous media. In another related work, the mathematical model of solid fuel Arrhenius combustion in a fixed-bed was analyzed by Olayiwola[9].Bhattacharyya *et al.*[10] studied analytically the solution for magnetohydrodynamic boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer. The unsteady boundary layer flow of a Casson fluid due to an impulsively started moving plate was considered by Mustafa *et al.* [11]. Recently, Mukhopadhyay*et al.*[12] investigated the steady boundary layer flow and heat transfer over a porous moving plate in the presence of thermal radiation. Makinde and Mhone[13] studied the heat transfer to MHD flow in a channel filled with aporous medium.Malapati&Polarapu [14] analyzed unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction. Chamkha and Ahmed [15] examined unsteady MHD heat and mass transfer by mixed convection flow in the forward stagnation region of a rotating sphere at different wall conditions. The effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a stretching in the presence of internal heat generation/absorption was studied by Elbashbeshy&Aldawody[16]. Talukdar[17] investigated the buoyancy and chemical reaction effects on MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and ohmic heating. Mohammed *et al.* [18] analyzed radiation and mass transfer effects on MHD oscillatory flow in a channel filled with porous medium in the presence of chemical reaction. The aim of the research is to establish an analytical solution capable of describing the concentration, temperature and velocity in the process of MHD Couette flow through a parallel plate.

2. Mathematical Formulation

Following Sayed-Ahmed *et al.* [1], the unsteady flow of a viscous, incompressible, non-conducting fluid through a channel with chemical reaction, thermal radiation, constant and variable pressure gradient in the presence of magnetic field is investigated. The flow is assumed to be laminar, incompressible and flows between two infinite horizontal plates located at $y = \pm h$ which extends from $x = -\infty$ to ∞ and from $z = -\infty$ to ∞.

The upper plate is suddenly set into motion and moves with a uniform velocity U_0 while the lower plate is kept stationary as shown in the diagram below. The upper plate is simultaneously subjected to a step change in temperature from T_1 to T_2. The upper and lower plates are kept at two constant temperatures T_2 and T_1 respectively with $T_2 > T_1$. The fluid flows between the two plates under the influence of an exponential decaying with time pressure gradient in the x-direction which is a generalization of a constant pressure gradient. A uniform suction from above and injection from below with constant velocity v_0 which are all applied at $t = 0$. The system is subjected to a uniform magnetic field B_0 in the positive y-direction and is assumed undisturbed as the induced magnetic field is neglected by assuming a small magnetic Reynolds number. The Hall effect is taken into consideration and consequently a z-component of the velocity is expected to arise.

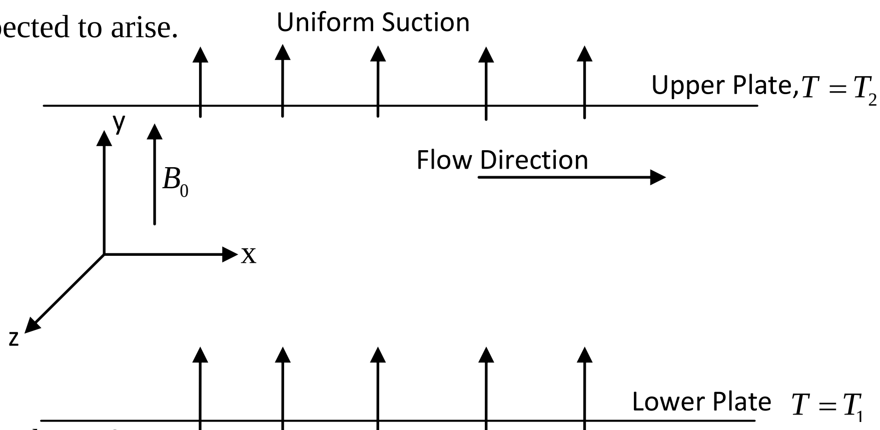

Figure 1: Schematic diagram of the problem

Based on the above assumptions,

$$v = ui + v_0 j + wk \qquad (1)$$

Introducing a Chapman-Rubesin viscosity law, with $w = 1$as shown in Olayiwola (2016) and using the condition at the lower plate, results in:

$$\mu = \frac{c\mu_1 T}{T_1} \qquad (2)$$

Where μ_1 is the Casson coefficient of viscosity.

Thus, the two components of the governing momentum equation in thedimensional form are as follows:

$$\rho \frac{\partial u}{\partial t} + \rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2}{(1+BiBe)^2 + Be^2}\left[(1+BiBe)u + Bew\right] -$$

$$\mu \frac{u}{k} + g\beta_T (T - T_1) + g\beta_C (C - C_1) \qquad (3)$$

$$\rho\frac{\partial w}{\partial t}+\rho v_0\frac{\partial w}{\partial y}=\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)-\frac{\sigma B_0^2}{(1+BiBe)^2+Be^2}\left[(1+BiBe)w-Beu\right]-\mu\frac{w}{k} \qquad (4)$$

The energy equation in dimensional form is given as

$$\left.\begin{aligned}&\rho C_P\frac{\partial T}{\partial t}+\rho C_P v_0\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right)+\mu\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\\&\frac{\sigma B_0^2}{(1+BiBe)^2+Be^2}\left[u^2+w^2\right]-\frac{1}{\rho C_P}\frac{\partial q}{\partial y}\end{aligned}\right\} \qquad (5)$$

The concentration equation in dimensional form is given as:

$$\rho\frac{\partial C}{\partial t}+\rho v_0\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial}{\partial y}\left(\mu\frac{\partial C}{\partial y}\right)+D_1\frac{\partial^2 T}{\partial y^2}-D_2\left(C_2-C_1\right) \qquad (6)$$

Subject to the initial and boundary conditions;

$$\left.\begin{aligned}&u(y,0)=0, && u(-h,t)=0, \quad u(h,t)=U_0\\&w(y,0)=U_0y(1-y), && w(-h,t)=0, \quad w(h,t)=0\\&T(y,0)=0, && T(-h,t)=T_1, \quad T(h,t)=T_2\\&C(y,0)=0, && C(-h,t)=C_1, \quad C(h,t)=C_2\end{aligned}\right\} \qquad (7)$$

Where ρ and μ are respectively the density and apparent viscosity of the fluid, σ is electric conductivity, β is Hall factor, Bi is ion slip parameter, $Be=\sigma\beta B_0$ is Hall parameter, c and k are the specific heat capacity and thermal conductivity of the fluidrespectively. Where u and w are components of velocities along and perpendicular to the plate in x and y directions respectively, σ is the electrical conductivity, β_T is the coefficient of volume expansion of the moving fluid, β_C is the coefficient of volumetric expansion with concentration, v is the kinematic viscosity, T is the temperature of the fluid, C is the concentration of the fluid, C_1 is theconcentration at infinity, D_1 the thermal diffusivity, D_2 the chemical reaction rate constant, C_P is the specific heat capacity at constant pressure. t is thetime, g is thegravitational force, μ_e is themagnetic permeability of the fluid, K is the porous media permeability coefficient, q is radiative heat flux, H_0 is the intensity of themagnetic field, $B_0 = \mu_e H_0$ is electromagnetic induction, τ_0 is yield stress, β is coefficient of volume expansion due to temperature and α is the mean radiation absorption coefficient.

To write the governing dimensional equations (3)-(6) with their corresponding boundary conditions (7) in non-dimensional form, we use the following dimensionless variables:

$$\bar{u} = \frac{u}{U_0}, \quad \bar{w} = \frac{w}{U_0}, \quad \bar{y} = \frac{y}{h}, \quad \bar{x} = \frac{x}{h}, \quad \bar{t} = \frac{tU_0}{h}, \quad \theta = \frac{T - T_1}{T_2 - T_1}$$

$$\phi = \frac{C - C_1}{C_2 - C_1}, \quad \bar{P} = \frac{P}{\rho U_0^2}, \quad \bar{\mu} = \frac{c\mu_1 T}{T_1} \tag{8}$$

and we obtain

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{\partial \bar{P}}{\partial x} + \frac{c}{\mathrm{Re}}\frac{\partial}{\partial y}\left((a\theta+1)\frac{\partial u}{\partial y}\right) - \frac{Ha^2}{\mathrm{Re}\left(1+BiBe\right)^2 + Be^2}\left[\left(1+BiBe\right)u + Bew\right] - \frac{Pc}{\mathrm{Re}}\left((a\theta+1)u\right) + Gr_\theta\theta + Gr_\phi\phi \tag{9}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = \frac{c}{\mathrm{Re}}\frac{\partial}{\partial y}\left((a\theta+1)\frac{\partial w}{\partial y}\right) - \frac{Ha^2}{\mathrm{Re}\left(1+BiBe\right)^2 + Be^2}\left[\left(1+BiBe\right)w - Beu\right] - \frac{Pc}{\mathrm{Re}}\left((a\theta+1)w\right) \tag{10}$$

$$\frac{\partial\theta}{\partial t}+S\frac{\partial\theta}{\partial y}=\frac{c}{\operatorname{Re}\Pr}\frac{\partial}{\partial y}\left[(a\theta+1)\frac{\partial\theta}{\partial y}\right]+\frac{cEc}{\operatorname{Re}}(a\theta+1)\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+$$

$$\frac{EcHa^2}{\operatorname{Re}\left[(1+BiBe)^2+Be^2\right]}\left[u^2+w^2\right]-Ra^2\theta \tag{11}$$

$$\frac{\partial\phi}{\partial t}+S\frac{\partial\phi}{\partial \bar{y}}=\frac{c}{Sc\operatorname{Re}}\frac{\partial}{\partial y}\left[(a\theta+1)\frac{\partial\phi}{\partial y}\right]+T_D\frac{\partial^2\theta}{\partial \bar{y}^2}-Kr\phi \tag{12}$$

Subject to the initial and boundary conditions

$$\left.\begin{array}{lll}
u(y,0)=0, & u(-1,t)=0, & u(1,t)=1 \\
w(y,0)=y(1-y), & w(-1,t)=0, & w(1,t)=0 \\
\theta(y,0)=0, & \theta(-1,t)=0, & \theta(1,t)=1 \\
\phi(y,0)=0, & \phi(-1,t)=0, & \phi(1,t)=1
\end{array}\right\} \tag{13}$$

Where

$$\left.\begin{array}{l}
\operatorname{Re}=\frac{\rho U_0 h}{\mu_1},\quad S=\frac{v_0}{U_0},\quad P=\frac{h^2\mu_1}{k},\quad Ha^2=\frac{\sigma B_0^2 h^2}{\mu_1},\quad Gr_\theta=\frac{g\beta_T(T_2-T_1)h}{\rho U_0^2}, \\
Gr_\phi=\frac{g\beta_C(C_2-C_1)h}{\rho U_0^2},\quad \Pr=\frac{\mu_1 c_p}{k},\quad Ec=\frac{U_0^2}{c_p(T_2-T_1)},\quad Ra^2=\frac{4\alpha^2 h}{\rho^2 C_p^2 U_0}, \\
Sc=\frac{U_0 h}{D},\quad T_D=\frac{D(T_2-T_1)}{h(C_2-C_1)U_0},\quad a=\frac{T_2-T_1}{T_1},\quad Kr=\frac{D_2 h}{\rho U_0},
\end{array}\right\} \tag{14}$$

3. Method of Solution

Since the boundary conditions are from -1 to 1, we first transform the boundary conditions to 0 to 1 using the transformation:

$$z = \frac{y+1}{2} \tag{15}$$

Let $0 < S << 1$ and $Ec = bS,\quad a = eS,\quad Be = fS,\quad Gr_{\theta} = gS,\quad Gr_{\phi} = hS$ such that

$$\left.\begin{aligned}
u(z,t) &= u_0(z,t) + Su_1(z,t) + \ldots \\
w(z,t) &= w_0(z,t) + Sw_1(z,t) + \ldots \\
\theta(z,t) &= \theta_0(z,t) + S\theta_1(z,t) + \ldots \\
\phi(z,t) &= \phi_0(z,t) + S\phi_1(z,t) + \ldots
\end{aligned}\right\} \tag{16}$$

Collecting like powers of S, we have for:

S^0 :

$$\left.\begin{aligned}
&\frac{\partial \theta_0}{\partial t} = \frac{c}{4\,\mathrm{Re}\,\mathrm{Pr}} \frac{\partial}{\partial z}\left[\frac{\partial \theta_0}{\partial z}\right] - Ra^2\theta_0 \\
&\theta_0(z,0) = 0, \quad \theta_0(0,t) = 0, \quad \theta_0(1,t) = 1
\end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} &\frac{\partial \phi_0}{\partial t} = \frac{c}{4Sc\,\mathrm{Re}}\frac{\partial}{\partial z}\left[\frac{\partial \phi_0}{\partial z}\right] + T_D\frac{\partial^2\theta_0}{\partial z^2} - Kr\phi_0 \\ &\phi_0(z,0)=0, \quad \phi_0(0,t)=0, \quad \phi_0(1,t)=1 \end{aligned}\right\} \qquad (18)$$

$$\left.\begin{aligned} &\frac{\partial w_0}{\partial t} = \frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial w_0}{\partial z}\right) - \frac{Ha^2}{\mathrm{Re}}[w_0] - \frac{Pc}{\mathrm{Re}}[w_0] \\ &w_0(z,0)=(2z-1)(2-2z), \quad w_0(0,t)=0, \quad w_0(1,t)=0 \end{aligned}\right\} \qquad (19)$$

$$\left.\begin{aligned} &\frac{\partial u_0}{\partial t} = \frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial u_0}{\partial z}\right) - \frac{Ha^2}{\mathrm{Re}}[u_0] - \frac{Pc}{\mathrm{Re}}(u_0) + \sigma \\ &u_0(z,0)=0, \quad u_0(0,t)=0, \quad u_0(1,t)=1 \end{aligned}\right\} \qquad (20)$$

$$\left.\begin{aligned} &\frac{\partial u_1}{\partial t} + \frac{1}{2}\frac{\partial u_0}{\partial z} = \frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{e\theta_0\partial u_0}{\partial z} + \frac{\partial u_1}{\partial z}\right) - \frac{Ha^2}{\mathrm{Re}}\left[u_0 Bif + u_1 + fw_0\right] - \frac{Pc}{\mathrm{Re}}(e\theta_0 u_0 + u_1) + \\ &S^{1}: g\theta_0 + h\phi_0 \\ &u_1(z,0)=0, \qquad u_1(0,t)=0, \qquad u_1(1,t)=0 \end{aligned}\right\} \qquad (21)$$

$$\left.\begin{aligned} &\frac{\partial \theta_1}{\partial t} + \frac{1}{2}\frac{\partial \theta_0}{\partial z} = \frac{c}{4\,\mathrm{Re}\,\mathrm{Pr}}\frac{\partial}{\partial y}\left[e\theta_0\frac{\partial \theta_0}{\partial z} + \frac{\partial \theta_1}{\partial z}\right] + \frac{bc}{4\,\mathrm{Re}}\left[\left(\frac{\partial u_0}{\partial y}\right)^2 + \left(\frac{\partial w_0}{\partial y}\right)^2\right] - \\ &\frac{bHa^2}{\mathrm{Re}}\left[(u_0)^2 + (w_0)^2\right] - Ra^2\theta_1 \\ &\theta_1(z,0)=0, \qquad \theta_1(0,t)=0, \qquad \theta_1(1,t)=0 \end{aligned}\right\} \qquad (22)$$

$$\left.\begin{aligned}
&\frac{\partial w_1}{\partial t}+\frac{1}{2}\frac{\partial w_0}{\partial z}=\frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(e\theta_0\frac{\partial w_0}{\partial z}+\frac{\partial w_1}{\partial z}\right)-\frac{Ha^2}{\mathrm{Re}}\left[w_0 Bif+w_1-fu_0\right]-\frac{Pc}{\mathrm{Re}}\left(e\theta_0 w_0+w_1\right)\\
&w_1(z,0)=0,\qquad w_1(0,t)=0,\qquad w_1(1,t)=0
\end{aligned}\right\}\tag{23}$$

$$\left.\begin{aligned}
&\frac{\partial \phi_1}{\partial t}+\frac{1}{2}\frac{\partial \phi_0}{\partial z}=\frac{c}{4Sc\,\mathrm{Re}}\frac{\partial}{\partial z}\left[e\theta_0\frac{\partial \phi_0}{\partial z}+\frac{\partial \phi_1}{\partial z}\right]+T_D\frac{\partial^2\theta_1}{\partial z^2}-Kr\phi_1\\
&\phi_1(z,0)=0,\qquad \phi_1(0,t)=0,\qquad \phi_1(1,t)=0
\end{aligned}\right\}\tag{24}$$

Eigenfunction Expansion Technique

Now, consider the problem (see Myint-U and Debnath, (1987))

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}+\alpha u+F(x,t)\\
&u(x,0)=f(x),\qquad u(0,t)=0,\qquad u(L,t)=0
\end{aligned}\right\}\tag{25}$$

For the solution of problem (24), we assume a solution of the form

$$u(x,t)=\sum_{n=1}^{\infty}u_n(t)\sin\frac{n\pi}{L}x\tag{26}$$

Where

$$u_n\left(t\right)=\int_0^t e^{\left(\alpha-k\left(\frac{n\pi}{L}\right)^2\right)(t-\tau)}F_n(\tau)d\tau+b_n e^{\left(\alpha-k\left(\frac{n\pi}{L}\right)^2\right)t} \quad (27)$$

$$F_n(t)=\frac{2}{L}\int_0^L F(x,t)\sin\frac{n\pi}{L}xdx \quad (28)$$

$$b_n(t)=\frac{2}{L}\int_0^L F(x)\sin\frac{n\pi}{L}xdx \quad (29)$$

Comparing equation (17) – (24) with the (25) we obtain the solutions to the velocity (primary and secondary), temperature, and concentration distributions as

$$\theta_0(\mathrm{z,t})=\mathrm{z}+\sum_{n=1}^{\infty}q_1\left(1-e^{-q_0 t}\right)\sin n\pi z \quad (30)$$

$$\phi_0(\mathrm{z,t})=z+v_2(\mathrm{z,t}) \quad (31)$$

$$w_0(\mathrm{z},t)=\sum_{n=1}^{\infty}q_{10}e^{-q_{11}t}Sinn\pi z \quad (32)$$

$$u_0\left(z,t\right)=z+\sum_{n=1}^{\infty}\frac{q_{12}}{q_{11}}\left(1-e^{-q_{11}t}\right)Sinn\pi z \quad (33)$$

$$u_1(\mathrm{z,t})=\sum_{n=1}^{\infty}u_5\left(t\right)Sinn\pi z \quad (34)$$

Where

$$u_5(t) = q_{16}\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right)\right) + \sum_{n=1}^{\infty} q_{27}P_4\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right) - te^{-q_{11}t}\right) +$$

$$\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{28}q_1P_4\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right) - te^{-q_{11}t} - \frac{1}{q_{11}-q_0}\left(e^{-q_0t} - e^{-q_{11}t}\right) - \frac{1}{q_0}\left(e^{(q_{11}-q_0)t} - e^{-q_{11}t}\right)\right) +$$

$$\sum_{n=1}^{\infty} q_{29}q_1\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right) - \frac{1}{q_{11}-q_0}\left(e^{-q_0t} - e^{-q_{11}t}\right)\right) +$$

$$q_{26}\left(\begin{array}{l} \sum_{n=1}^{\infty} q_4\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right) - \frac{1}{q_{11}-q_2}\left(e^{-q_2t} - e^{-q_{11}t}\right)\right) + \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_5P_1\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right) - te^{-q_{11}t} - \frac{1}{q_{11}-q_2}\left(e^{-q_2t} - e^{-q_{11}t}\right)\right) - \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_5P_2\left(\frac{1}{q_{11}-q_0}\left(e^{-q_0t} - e^{-q_{11}t}\right) - \frac{1}{q_{11}-q_2}\left(e^{-q_2t} - e^{-q_{11}t}\right)\right) \end{array}\right)$$

$$\theta_1(z,t) = \sum_{n=1}^{\infty} u_6(t)\, Sinn\pi z \qquad (35)$$

Where

$$u_6(t)=\begin{pmatrix} \sum_{n=1}^{\infty} q_1 q_{30}\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-te^{-q_0 t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_1^2 q_{31}\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-2te^{-q_0 t}-\frac{1}{q_0}\left(e^{-2q_0 t}-e^{-q_0 t}\right)\right)+ \\ q_{32}\frac{1}{q_0}\left(1-e^{-q_0 t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{10}^2 q_{33}\left(\frac{1}{q_0-2q_{11}}\left(e^{-2q_{11} t}-e^{-q_0 t}\right)\right)+ \\ \sum_{n=1}^{\infty} q_{34} P_4\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-\frac{1}{q_0-q_{11}}\left(e^{-q_{11} t}-e^{-q_0 t}\right)\right)+ \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{35} P_4^2\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-\frac{2}{q_0-q_{11}}\left(e^{-q_{11} t}-e^{-q_0 t}\right)+\frac{1}{q_0-2q_{11}}\left(e^{-2q_{11} t}-e^{-q_0 t}\right)\right) \end{pmatrix}$$

$$w_1(z,t)=\sum_{n=1}^{\infty} u_7(t)\, Sinn\pi z \tag{36}$$

Where

$$u_7(t)=\begin{pmatrix} q_{10} q_{36}\left(te^{-q_{11} t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{37} q_1 q_{10}\left(te^{-q_{11} t}+\frac{1}{q_0}\left(e^{-(q_{11}-q_0)t}-e^{-q_{11} t}\right)\right)+q_{38}\left(\frac{1}{q_{11}}\left(1-e^{-q_{11} t}\right)\right)+ \\ \sum_{n=1}^{\infty} q_{14} P_4\left(\frac{1}{q_{11}}\left(1-e^{-q_{11} t}\right)+te^{-q_{11} t}\right) \end{pmatrix}$$

$$\phi_1(z,t)=\sum_{n=1}^{\infty} u_8(t)\, Sinn\pi z \tag{37}$$

Where

$$
u_8(t)=\left(\begin{array}{l}
q_{39}\left(\begin{array}{l}
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{43}\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{44}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{45}\frac{t\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{46}\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{47}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\\
2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{48}\frac{t\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{49}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{50}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\\
\sum_{n=1}^{\infty}q_{51}\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{52}\frac{\left(e^{-2q_{11}t}-e^{-q_2t}\right)}{q_2-q_0}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{53}\frac{\left(e^{-2q_{11}t}-e^{-q_2t}\right)}{q_2-q_0}+\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{54}\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{55}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{56}\frac{\left(e^{-q_{11}t}-e^{-q_2t}\right)}{q_2-q_{11}}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{57}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{58}\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}\right)-2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{59}\frac{\left(e^{-q_{11}t}-e^{-q_2t}\right)}{q_2-q_{11}}+2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{60}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{61}\frac{\left(e^{-2q_{11}t}-e^{-q_2t}\right)}{q_2-2q_{11}}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{62}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}
\end{array}\right)\\
q_{40}\frac{\left(1-e^{-q_2t}\right)}{q_2}+\\
q_{41}\left(\begin{array}{l}
-\sum_{n=1}^{\infty}q_4\frac{\left(1-e^{-q_2t}\right)}{q_2}+\sum_{n=1}^{\infty}q_4te^{-q_2t}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_1\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_1te^{-q_2t}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_2\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_2te^{-q_2t}
\end{array}\right)\\
q_{42}\left(\begin{array}{l}
-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_1q_4\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-te^{-q_2t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_1\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-te^{-q_2t}\right)-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_2\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-te^{-q_2t}\right)+\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_1q_4\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\frac{\left(e^{-(q_2+q_0)t}-e^{-q_2t}\right)}{q_0}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_1\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\frac{\left(e^{-(q_2+q_0)t}-e^{-q_2t}\right)}{q_0}\right)+\\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_2\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\frac{\left(e^{-(q_2+q_0)t}-e^{-q_2t}\right)}{q_0}\right)
\end{array}\right)
\end{array}\right)
$$

Therefore the solutions to the governing equations are given as:

$$\theta(\mathrm{z,t}) = \mathrm{z} + \sum_{n=1}^{\infty} q_1\left(1-e^{-q_0 t}\right)\sin n\pi z + \sum_{n=1}^{\infty} u_6\left(t\right) Sinn\pi z \tag{38}$$

$$\phi(\mathrm{z,t}) = z + v_2(\mathrm{z,t}) + s\sum_{n=1}^{\infty} u_8\left(t\right) Sinn\pi z \tag{39}$$

$$w(\mathrm{z},t) = \sum_{n=1}^{\infty} q_{10} e^{-q_{11}t} Sinn\pi z + s\sum_{n=1}^{\infty} u_7\left(t\right) Sinn\pi z \tag{40}$$

$$u\left(z,t\right) = z + \sum_{n=1}^{\infty} \frac{q_{12}}{q_{11}}\left(1-e^{-q_{11}t}\right) Sinn\pi z + s\sum_{n=1}^{\infty} u_5\left(t\right) Sinn\pi z \tag{41}$$

4 Results and Discussions

The system of partial differential equations describing unsteady couette flow of an electrically conducting incompressible fluid bounded by two parallel non conducting porous plates (is)are solved analytically using eigenfunction expansion method. The analytical solutions of the governing equations are computed and presented graphically with the aid of a computer symbolic algebraic package MAPLE 17 for the values of the following parameters:

$\mathrm{Re}=1,\quad Ra^2=1,\quad s=0.1,\quad \mathrm{Pr}=0.71,\quad Ha^2=1,\quad Kr=0.5,\quad Sc=0.22,$

$Bi=1,\quad Be=1,\quad a=0.1,\quad c=0.2,\quad P=1,\quad T_D=0,\quad Ec=0.01,$

$\mathrm{Gr}_\theta=0.2,\quad Gr_\phi=0.2,\quad \sigma=2$

The figures 2-12 Explains the graphs of primary and secondary velocities, temperature and concentration against different dimensionless parameters.

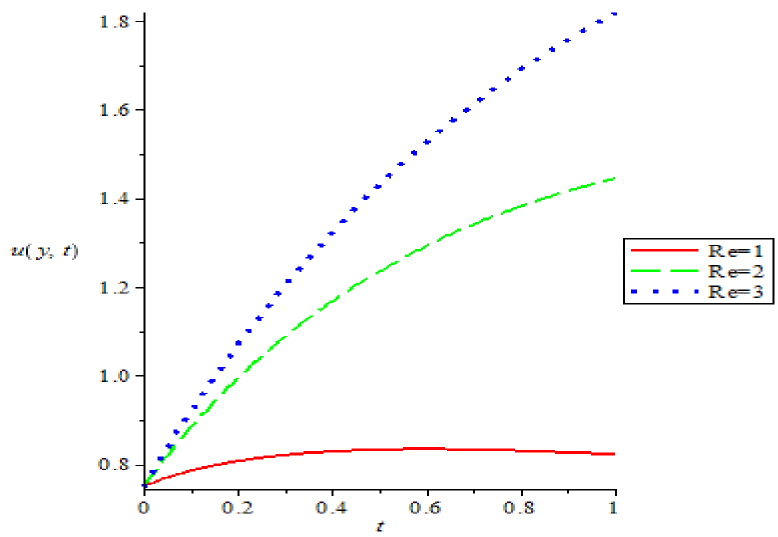

Figure 2 Relationship between Primary velocity and time for different values of Re

Figure 2 presents the graph of primary velocity with time for different values Reynolds number (Re). It is observed that primary velocity increases with time and also increases as theReynolds number increases.

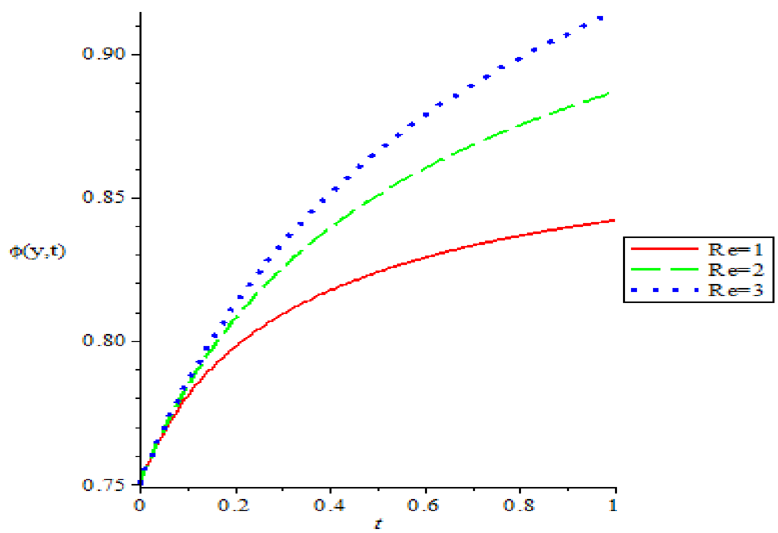

Figure 3 Relationship between concentration and time for different values of Re

Figure 3 presents the graph of concentration profile with time t for different values of theReynolds number. It is observed that the concentration profile increases with time and also, increases as theReynolds number increases

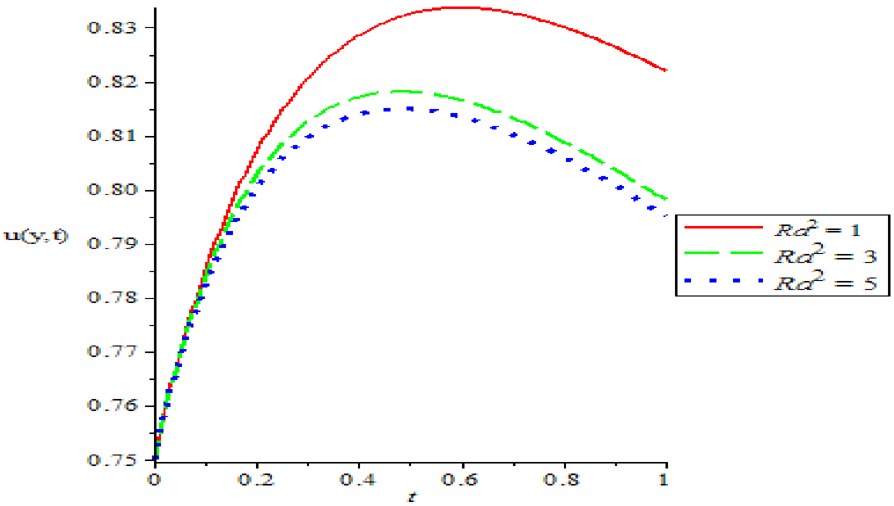

Figure 4 Relationship between primary velocity and time for different values of Ra

Figure 4 shows the influence of radiation on the primary velocity profile. It is evident that the primary velocity increases with time. Also, anincrease in the radiation parameter is found to decelerate the primary velocity of the flow.

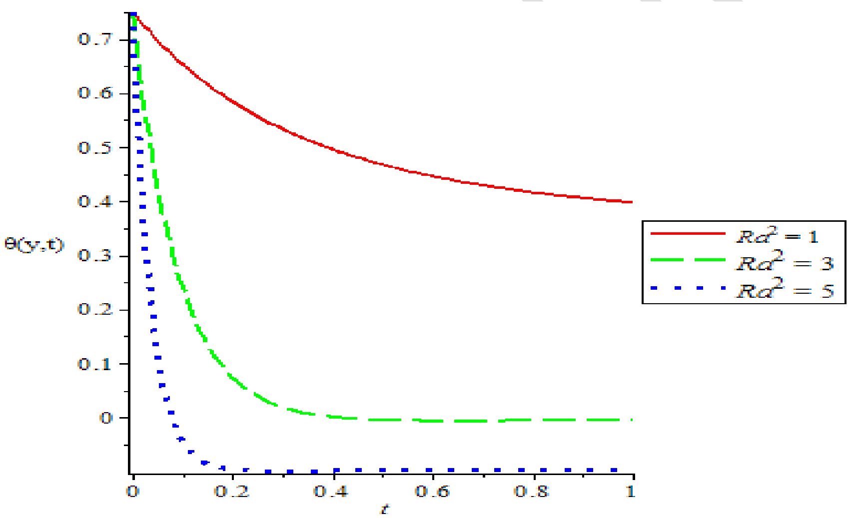

Figure 5 Relationship between temperature and time for different values of Ra^2

Figure 5 displays the effect of thethermal radiation parameter on the thermal profile of the flow with time t. It is observed that the flow field suffers a decrease in temperature as theradiation parameter increases while as radiation parameter the temperature decreases with time t.

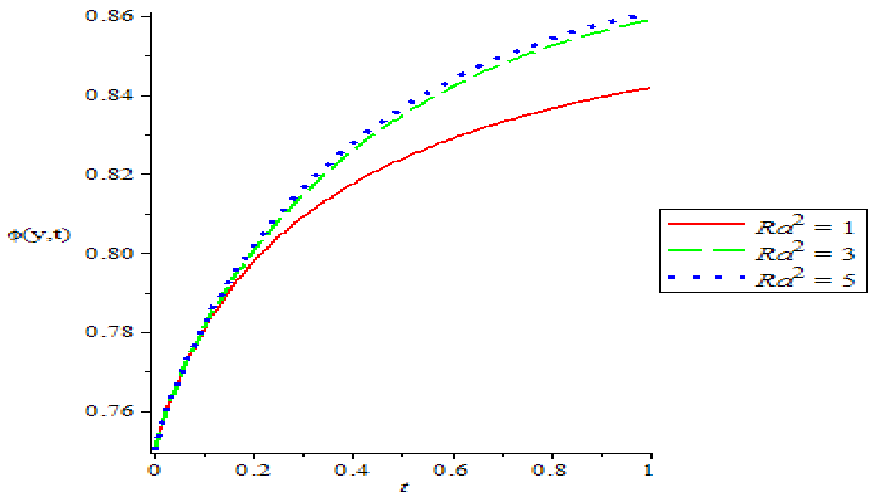

Figure 6 Relationship between concentration and time for different values of Ra^2

Figure 6 depicts the graph of concentration with time t for different values of theradiation parameter. It is evident that concentration increases with time and also increases as radiation increases.

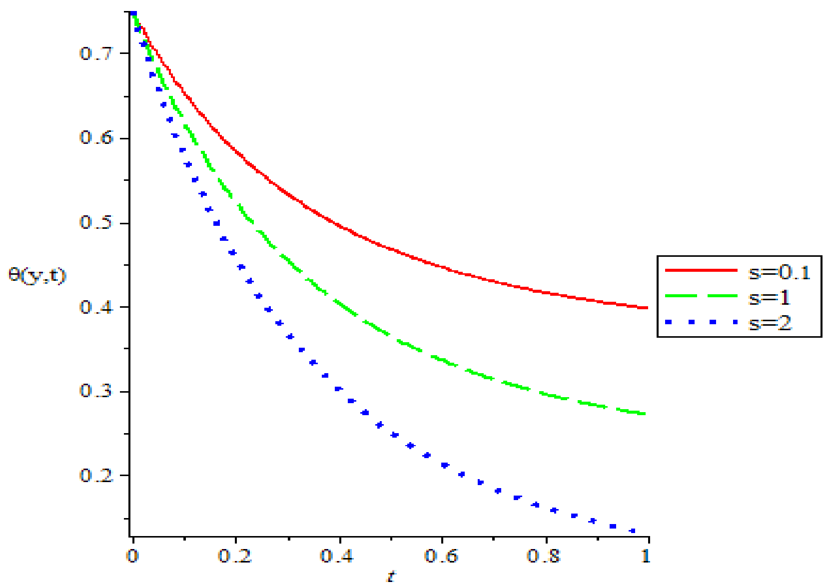

Figure 7 Relationship between temperature and time for different values of S

Figure 7 illustrates the graph of temperature with time for different values of thesuction parameter. It is seen that temperature decreases with time and also decreases as thesuction parameter increases.

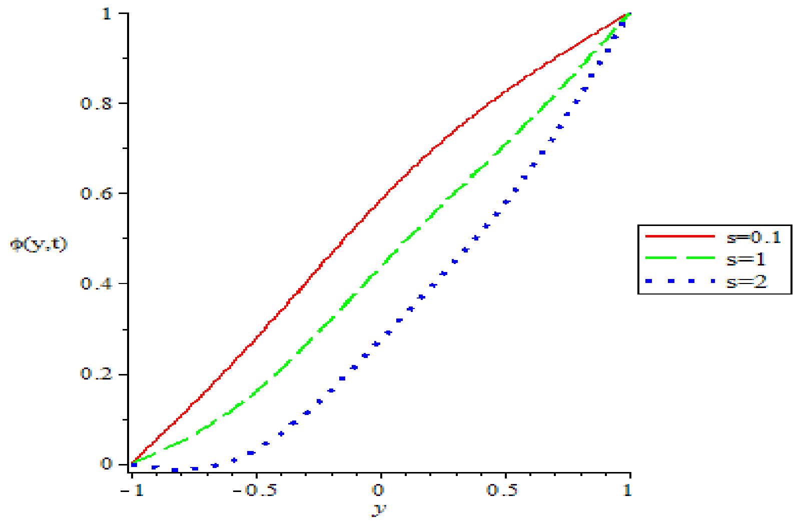

Figure 8 Relationship between concentration and distance for different values of S

Figure 8 presents the effect of thesuction parameter on concentration along distance y. it is observed that anincrease in suction parameter leads to adecrease in concentration alongwith distance while concentration is observed to increase along distance y.

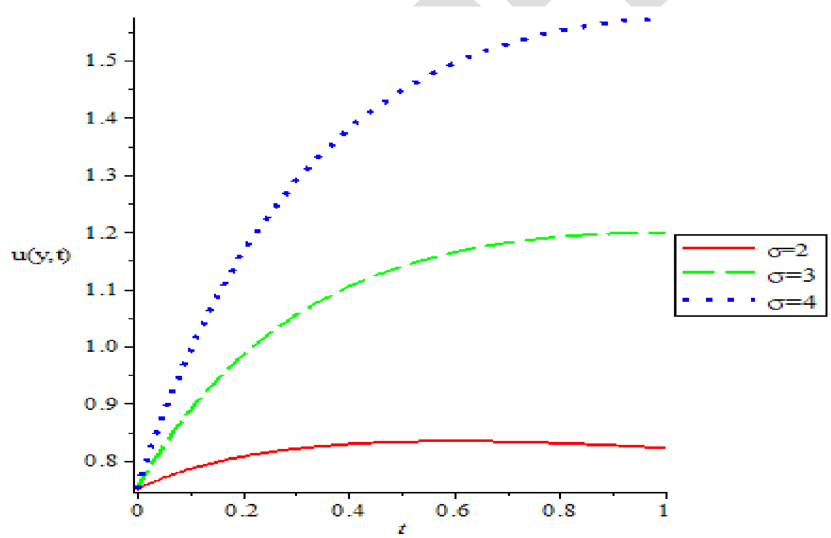

Figure 9 Relationship between primary velocity and time for different values of σ

Figure 9 shows the influence of thepressure gradient on theprimary velocity with time. It is observed that anincrease in pressure gradient leads to anincrease in primary velocity. Also, primary velocity is found to increase with time.

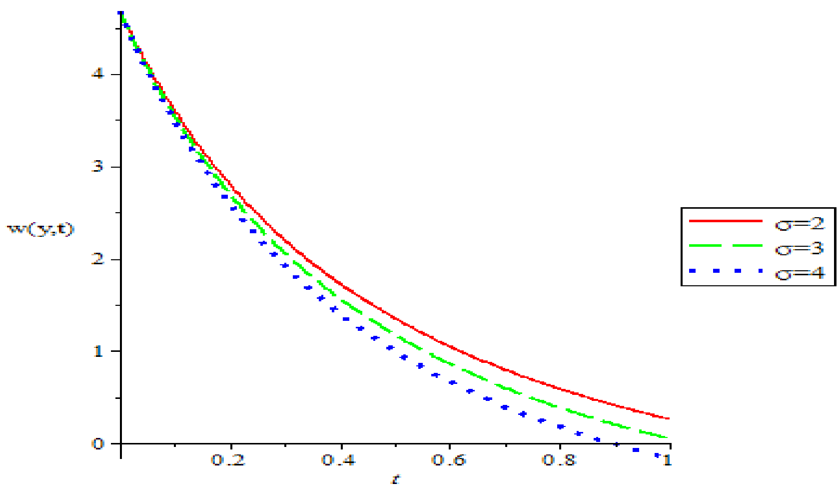

Figure 10 Relationship between secondary velocity and time for different values of σ

Figure 10 shows the effect of thepressure gradient on thesecondary velocity with time t. It is observed that anincrease in pressure gradient leads to adecrease in thesecondary velocity of the fluid while the secondary velocity is observed to decrease with time t.

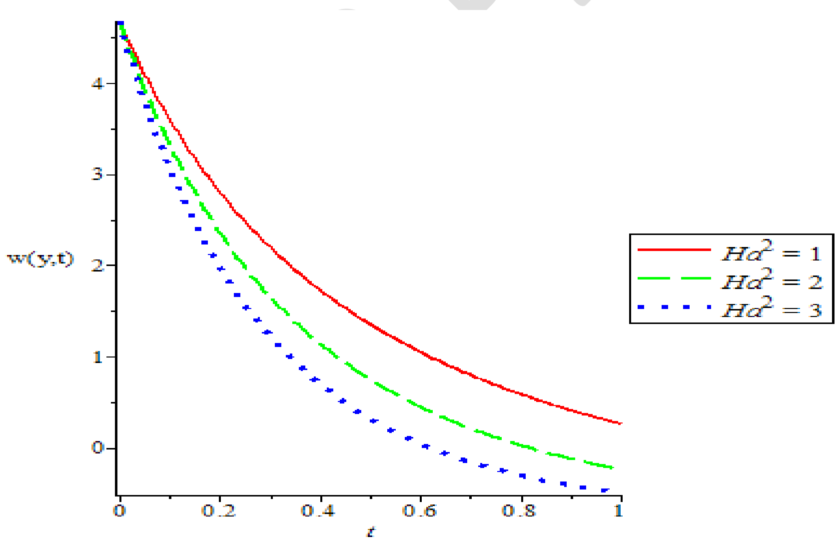

Figure 11 Relationship between secondary velocity and time for different values of Ha^2

Figure 11 presents the graph of secondary velocity with time t for different values of Hartman number. It is observed that anincrease in theHartman number leads to adecrease in secondary velocity. This is due to the retarding Lorentz force which acts in theopposite direction of the fluid flow when themagnetic field is applied. This type of resisting force, slows down the velocity as shown in the figure.

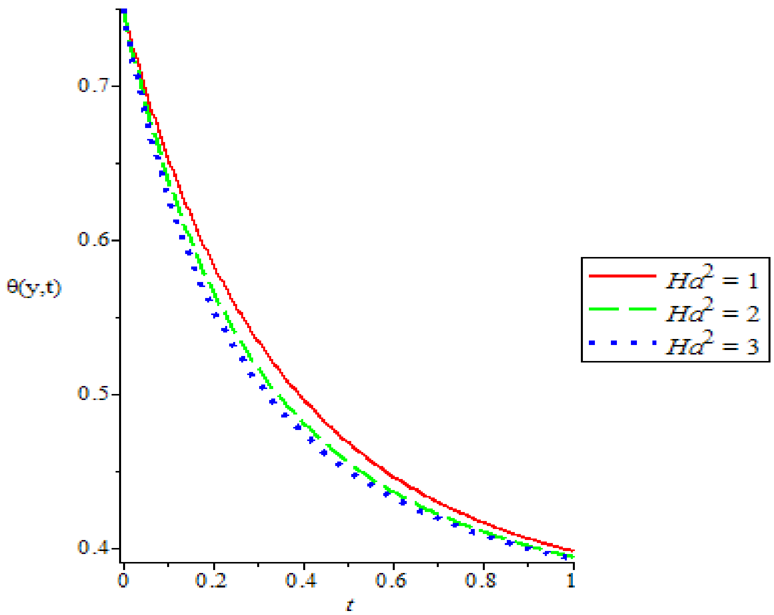

Figure 12 Relationship between temperature and time for different values of Ha^2

Figure 12 shows the effect of Hartman number with time t on thetemperature profile. It is observed that anincrease in Hartman number leads to adecrease in temperature. Also, the temperature profile is observed to decrease with time.

5. Conclusion

For constant pressure gradient, the unsteady MHD Couette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates under the influence of thermal radiation and chemical reaction is investigated. A uniform suction and injection are applied perpendicular to the plate. The transformed conservation equations are solved analytically subject to physically appropriate boundary conditions by using theEigenfunction expansion technique. From the results obtained, we can conclude that:

1. The increase in Hartman number leads to adecrease in velocity. This is due to the retarding Lorentz force which acts in theopposite direction of the fluid flow when themagnetic field is applied.
2. Concentration profile increases with time and also, increases as theReynolds number increases.
3. The increase in the radiation parameter is found to decelerate the velocity of the flow.
4. The flow field suffers a decrease in temperature as theradiation parameter increases while as radiation parameter the temperature profile is observed to decreases with time t.

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