# Analysis of Effect of Inclined Magnetic Field on MHD Boundary Layer Flow over a porous Exponentially Stretching Sheet subject to Thermal Radiation

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## ABSTRACT

In this study, MHD boundary layer flow of a viscous incompressible fluid over an exponentially stretching sheet with an inclined magnetic field in presence of thermal radiation is analyzed. The continuity, momentum and energy equations governing the fluid motion are obtained. They are then transformed into a system of nonlinear ordinary differential equations using similarity transformations. The resulting nonlinear ordinary differential equations are then changed to a system of first order ordinary differential equations in order to obtain the solution numerically by collocation method. The effects of magnetic field, angle of inclination, radiation, Prandtl number and the exponential stretching of the sheet on velocity and temperature of the fluid are discussed.

*Keywords: Inclined Magnetic field, Hydrodynamic boundary layer flow*

## 1. INTRODUCTION

The flow of fluids over a stretching sheet is considered as very important phenomena to study due to its wide application in industrial processes such as in the production of polymer sheets, filaments and wires. The assumption is that the stirring sheet move on its own plane and the stretched surface interacts with ambient fluid both impulsibly and thermally. The presence of magnetic field on the MHD boundary layer flow of a viscous incompressible fluid over an exponentially porous stretching sheet has some negative effects on the velocity of fluid flow. When the magnetic field is perpendicular to the stretching sheet the velocity of the fluid is suppressed. Suppression of the velocity of the fluid by magnetic field can be controlled by inclining the magnetic field, which allows for the variation of the angle of inclination. This makes systems with inclined magnetic field have velocity of fluid on them increased due to minimization of suppression. The concept of boundary layer was brought by Ludwig Prandtl [1874-1953] in his paper of 1904. His work on boundary layer formed the basis for future work on skin friction, heat transfer and separation. Due to drag the fluid velocity immediately adjacent to the surface is zero and the fluid layer next to the surface becomes attracted to the surface. That is, wets the surface. This condition is known as 'no slip condition'.[1]investigated and discussed the boundary layer flow over a surface by considering the numerical solutions of laminar boundary-layer behavior on a moving continuous flat surface.[2]validated Sakiadis work by considering the experimental and analytical behavior of this problem. [3]investigated the flow for linearly and exponentially stretching sheet for a steady two-dimensional viscous flow. [4]talked about free convective on a vertical stretching surface. [5]analyzed heat transfer over an exponentially stretching continuous surface with suction and similarity solutions of the laminar boundary layer. Equations of heat and flow in a quiescent fluid driven by exponentially stretching surface with suction were obtained. [6]presented his work on viscoelastic MHD flow, heat and mass transfer over stretching sheet with dissipation of energy and stretch work. Heat transfer over a stretching surface with variable heat flow in micro polar fluids was done by[7].
[8]discussed the effects of thermal radiation on the boundary layer flow of the Jeffrey fluid over an exponentially stretching surface. Also [9]discussed the MHD boundary layer flow of a Casson

fluid over an exponentially permeable stretching sheet. The literature available study the boundary layer flow over a stretching surface where the velocity of this surface is linearly proportional to the distance from the origin but in the real sense the stretching of plastic sheet may not necessarily be linear.effects of thermal radiation and magnetic filed on unsteady mixed convection flow and heat transfer over an exponentially stretching surface with suction in the presence of internal heat generation/absorptionwere investigated[10].[11]extended the flow and heat transfer analysis in boundary layer over an exponentially stretching sheet with radiation embedded in stratified medium.
The purpose of this present work is to analyze theeffect of inclined magnetic field on MHD boundary layer flow over a porous exponentially stretching sheet subject to thermal radiation.

## 2. MATHEMATICAL FORMULATION

In this study a magnetic field, $B$ inclined at an angle $\alpha$ to the horizontal flat sheet and how it influences the flow is considered. To achieve this we consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y=0 in a densely saturated porous medium with a non–uniform permeability $k$

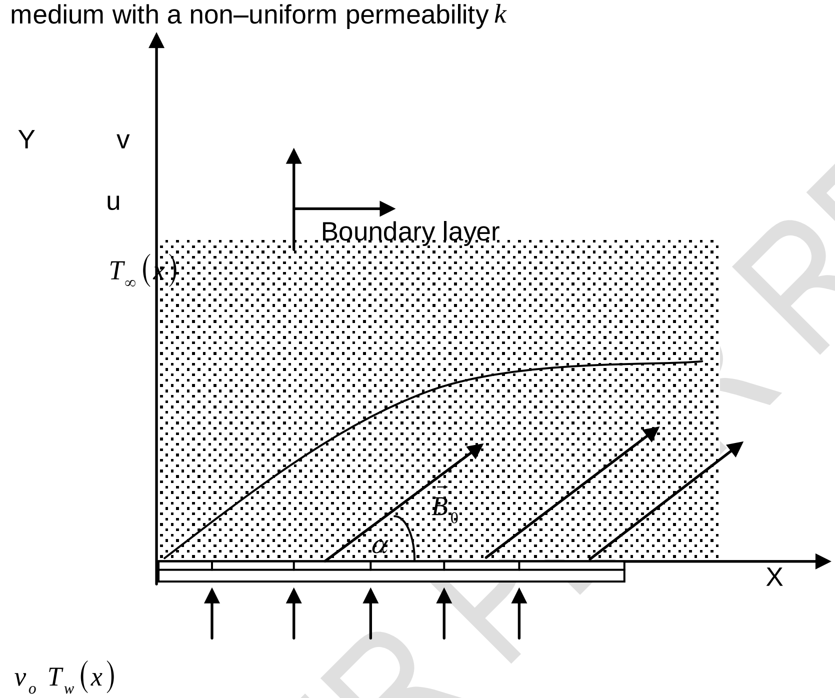


**Figure 1: Sketch of the physical problem.**

Assuming that the fluid flow is restricted to y>0, two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. These two equal and opposite forces cause a symmetric boundary at the centre of the porous medium (see fig 1). A variable magnetic field $\vec{B}(x) = \vec{B}_0 e^{\frac{Nx}{2L}}$ is applied inclined at an angle $\alpha$ to the sheet where from figure 1, $\vec{B}_0$ which is a constant is expressed as $\vec{B}_0 = (B_0 \cos\alpha, B_0 \sin\alpha, 0)$. Then a two dimensional steady flow in a magnetic field $\vec{B}$, induces an electric current density $\vec{J}$. The induced current creates forces on the liquid and changes the magnetic field. Each volume of the fluid having a magnetic field $\vec{B}$ experiences aLorentz force given by $\vec{J} \times \vec{B} = -\sigma u B^2 \sin^2 \alpha$ .

Let the sheet of temperature $T_w(x)$ be embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ , where $T_w(x) > T_\infty(x)$ .we define $T_w(x) = T_0 + be^{\frac{Nx}{2L}}$ and $T_\infty(x) = T_0 + ce^{\frac{Nx}{2L}}$ where $T_0$ is the reference temperature, $b > 0, c \ge 0$ are constants. The continuity, momentum and energy equations governing this flow are given by the equations below

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{k}u - \sigma\frac{B^2 \sin^2 \alpha}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions for the differential equations above are

$$u = U \quad , v = V(x) \quad , T = T_w(x) \quad \text{at } y = 0 \tag{4}$$

$$u \to 0, \quad T = T_\infty(x) \quad \text{as } y \to \infty \tag{5}$$

Where $u$ and $v$ are the components of velocity in the x and ydirection respectively, $\upsilon = \frac{\mu}{\rho}$ is the kinematics viscosity, $\rho$ is the fluid density, $\mu$ is the coefficient of fluid viscosity, $c_p$ is the specific heat capacity at constant pressure and $\kappa$ is the thermal conductivity of the fluid. $U = U_0 e^{\frac{Nx}{L}}$ Is the stretching velocity, $U_0$ is the reference velocity $V(x) < 0$ is velocity of blowing and $V(x) = V_0 e^{\frac{Nx}{2L}}$ , is a special type of velocity at the wall considered. $V_0$ is the initial strength of injection (blowing)

### 2.1 Similarity transformation

By introducing the similarity transformation variables

$$\eta = \sqrt{\frac{U_0}{2\upsilon L}}\, e^{\frac{Nx}{2L}} y \quad , \quad u = U_0 e^{\frac{Nx}{2L}}$$
$$v = -N\sqrt{\frac{\upsilon U_0}{2L}} e^{\frac{Nx}{2L}}\left(f(\eta) + \eta f'(\eta)\right) \quad , \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_0} \tag{6}$$

Using them in equations 2 and 3, we transform the governing equations as below
For equation 2

$$f''' - 2Nf'^2 + Nff'' - (k_1 + 2M^2 \sin^2 \alpha) f' = 0 \tag{7}$$

Where $k_1 = \frac{2\upsilon L}{k_0 U_0}$ and $M = B_0 \frac{\sqrt{L\sigma}}{\rho U_0}$

Then equation 3 reduces to

$$\{1 + \frac{4}{3}Rd\}\, \theta'' + N\Pr\{f\theta' - \theta f'\} - N\Pr St f' = 0 \tag{8}$$

Where $\Pr = \dfrac{\mu c_p}{\kappa}$, $St = \dfrac{c}{b}$ and $Rd = \dfrac{4\sigma T_\infty^3}{k^* \kappa}$

The boundary conditions associated with these equations are transformed as below using the boundary conditions (4) and (5) with the similarity transformations (6)

$$f' = 1, \quad f = S, \quad \theta = 1 - St \quad \text{at}\ \eta = 0 \tag{9}$$

$$f' \to 0, \quad \theta \to 0 \quad \text{as}\ \eta \to \infty \tag{10}$$

Where the prime denotes differentiation with respect to $\eta$ ,

$M = B_o \dfrac{\sqrt{L\sigma}}{\rho U_0}$ is the magnetic parameter, $S = \dfrac{v_0}{\sqrt{\dfrac{U_0 \nu}{2L}}} < 0)$ is the blowing parameter, $St = \dfrac{c}{b}$ is the stratification parameter and $\Pr = \dfrac{\mu c_p}{\kappa}$ is the Prandtl number, $Rd = \dfrac{4T_\infty^3 \sigma}{3kk^*}$ is the radiative parameter $N$ ,is the exponentially stretching sheet parameter and $k_1 = \dfrac{2\nu L}{k_0 U_0}$ is the permeability parameter. For $St > 0$ , gives a stably stratified environment, while $St = 0$ implies an unstratified environment.

Lai and Kulacki (1991) discussed about the skin friction coefficient and the Nusselt number which are vital physical quantities in our problem. These quantities are defined as

Skin friction coefficient,

$$c_f = -\frac{U_0}{x\sqrt{2}} \left( \frac{\partial u}{\partial y} \right)_{y=0} = f''(0) \tag{11}$$

Nusselt number,

$$Nu = x \left( \frac{\partial T}{\partial y} \right)_{y=0} = -\theta'(0) \tag{12}$$

**2.2 Conversion of higher order ODEs to first order ODEs from**

In the solution technique we obtain a system of first order ordinary differential equations from the determined higher order ordinary differential equations.

For

$$f''' - 2Nf'^2 + Nff'' - (k_1 + 2M^2 \sin^2 \alpha) f' = 0 \text{ and}$$

$$\theta'' + \frac{N \Pr}{\left(1 + \frac{4}{3} Rd\right)} \{ f\theta' - \theta f' \} - \frac{N \Pr St}{\left(1 + \frac{4}{3} Rd\right)} f' = 0$$

Let $u_1 = f$, $u_2 = f'$, $u_3 = f''$, $u_4 = \theta$, $u_5 = \theta'$ To obtain system below

$$\begin{aligned}
u_1' &= u_2 \\
u_2' &= u_3 \\
u_3' &= 2Nu_2^2 - Nu_1 u_3 + \left(k_1 + 2M^2 \sin^2 \alpha\right) u_2 \\
u_4' &= u_5 \\
u_5' &= \frac{N \Pr}{\left(1 + \frac{4}{3} Rd\right)} (u_4 u_2 - u_1 u_5) + \frac{N \Pr St}{\left(1 + \frac{4}{3} Rd\right)} u_2
\end{aligned} \tag{13}$$

With boundary conditions

$$u_1 = S, \quad u_2 = 1, \quad u_3 = 0, \quad u_4 = 1 - St, \quad u_5 = 0 \quad at \quad \eta = 0 \tag{14}$$

$$u_2 \to 0, \quad u_4 \to 0 \quad as \quad \eta \to \infty \tag{15}$$

### 2.3 Numerical solution

To solve the system of equations above numerically, we use the collocation method .For this method to apply, the system of equations above is written in vector form as below

$$\vec{u}' = \vec{g}(\eta, \vec{u}, \vec{p}) \quad for\ 0 \le \eta < \infty \tag{16}$$

Where $\vec{u} = (u_1, u_2, u_3, u_4, u_5)^T$ , $\vec{g} = (g_1, g_2, g_3, g_4, g_5)^T$ and $\vec{p}$ is a vector of unknown parameters. $\vec{g}$ Takes the values

$$g_1 = u_2$$

$$g_2 = u_3$$

$$g_3 = 2Nu_2^2 - Nu_1 u_3 + (k_1 + 2M^2 \sin^2 \alpha) u_2$$

$$g_4 = u_5$$

$$g_5 = \frac{N \Pr}{\left(1 + \frac{4}{3} Rd\right)}(u_4 u_2 - u_1 u_5) + \frac{N \Pr St}{\left(1 + \frac{4}{3} Rd\right)} u_2$$

Equation (16) is solved subject to boundary conditions, see 14, 15

$$\vec{h}(\vec{u}(0), \vec{u}(\infty), \vec{p}) = 0 \tag{17}$$

For simplicity we suppress $\vec{p}$ in equation (16) to get an approximate solution $\vec{S}(\eta)$ to $\vec{u}(\eta)$, which is a continuous function that is a cubic polynomial on each subinterval $(\eta_n, \eta_{n+1})$ of a mesh $0 = \eta_0 < \eta_1 < .......... ..... < \eta_N = \infty$ .This approximate solution satisfies:

(a) The boundary conditions

$\vec{h} = (S(0), S(\infty)) = 0$ (18)(b) differential equations (collocates) at both ends and the midpoint of each of the following subinterval

$$\vec{S}'(\eta_n) = \vec{g}(\eta_n, \vec{S}(\eta_n))$$

$$\vec{S}'\left( (\eta_n + \eta_{n+1})/2 \right) = \vec{g}\left( (\eta_n + \eta_{n+1})/2, \vec{S}(\eta_n + \eta_{n+1})/2 \right)$$

$$\vec{S}'(\eta_{n+1}) = \vec{g}(\eta_{n+1}, \vec{S}(\eta_{n+1}))$$

These conditions give a system of nonlinear algebraic equations for the coefficients defining $\vec{S}(\eta)$ ,which is a cubic polynomial approximating the solution $\vec{u}(\eta)$ over the whole interval $[0, \infty)$ In collocation, these nonlinear equations are solved iteratively by linearization subject to the conditions

$$\left\| \vec{u}(\eta) - \vec{S}(\eta) \right\| \le Ch^4 \tag{19}$$

Where $h$ the maximum of the step is sizes $h_n = \eta_{n+1} - \eta_n$ for $n = 1,2,.......... .....N$ and $C$ is a constant. For the initial guess in collocation method, we note that the continuity of $\vec{S}(\eta)$ on $[0, \infty)$ and collocation at the ends of each subinterval imply that $\vec{S}(\eta)$ also has a continuous derivative on $[0, \infty)$ .Therefore for an approximate $\vec{S}(\eta)$ ,a residue $\vec{r}(\eta)$ in the above system of ODEs is computed as below

$$\vec{r}(\eta) = \vec{S}(\eta) - \vec{g}(\eta, \vec{S}(\eta)) \tag{20}$$

Similarly, the residual in the boundary conditions is obtained from (20) above. If the residuals are uniformly small, then $\vec{S}(\eta)$ is the required approximation of the exact solution $\vec{u}(\eta)$ .The idea behind this is to ensure that the residuals is minimized by making sure that the condition (19) is met at each point.

## 3. RESULTS AND DISCUSSION

The importance of this section is to analyze the effect of various physical parameters on the velocity and temperature profiles on the fluid flow. The results are presented graphically in figures 2-10 followed by a detailed discussion on the interpretation of the same for parameters such as, Exponentialstretching $(N)$, Stratification $(St)$, Magnetic $(M)$, Radiative $(Rd)$, Angle of inclination $(\alpha)$, andPrandtl number $(\Pr)$ which we chose to range between 4 and 6 for surfactants (Drag reducing agents).Also various values for skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ were obtained for each parameter and tabulated in table 1.

### 3.1: Effects of variation of magnetic parameter on velocity and temperature profiles

Fig.2 shows the effect of magnetic parameter $M$ on the velocity profile for the fluid flow, when other parameters are kept constant. According to the referred figure, the velocity decreases as values of $M$ increases. This is due to the fact that increase in the magnetic field results to an increase in Lorentz force. This offers more resistance to the motion of the fluid and thus the velocity of the fluid is reduced. In *fig.3* the temperature increases with increase in magnetic parameter $M$ .This is because larger values of magnetic parameter correspond to an increase in Lorentz force which is a resistive force. This resistive force results to an increase in thermal boundary layer which increases the temperature profile.

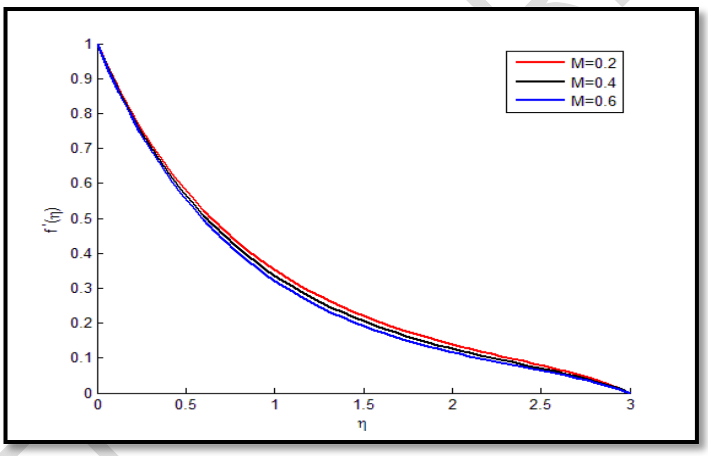


**Figure 2: Velocity profile for different values of magnetic parameter**

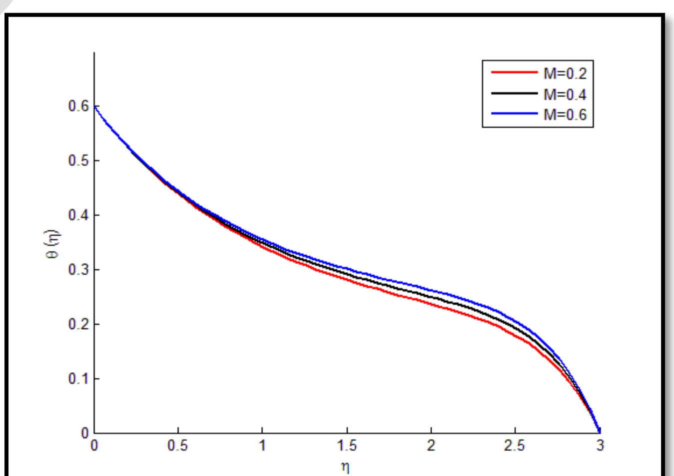

**Figure 3: Temperature profile for various values of magnetic parameter**

### 3.2: Effects of variation of angle of inclination on velocity and temperature profiles

*Fig.4* represents the effect angle of inclination $\alpha$ on velocity profile. It is clear that the velocity profile decreases by increasing the values of angle of inclination $\alpha$ .This can be attributed to the fact that an increase in angle of inclination, results to an increase in magnetic field effect on the fluid which in turn increases the Lorentz force resulting to decreased velocity profile. According to the result obtained it is clear that maximum resistance is experienced by the fluid particles when the angle is $\frac{\pi}{2}$.

*Fig.5* shows the variation of angle of inclination $\alpha$ on temperature profile. According to the figure referred, temperature profile is higher for larger values of angle $\alpha$ . This is due to the fact that higher values of angle $\alpha$ corresponds to larger magnetic field which opposes motion. This makes the thermal boundary layer to increase therefore increasing the temperature profile

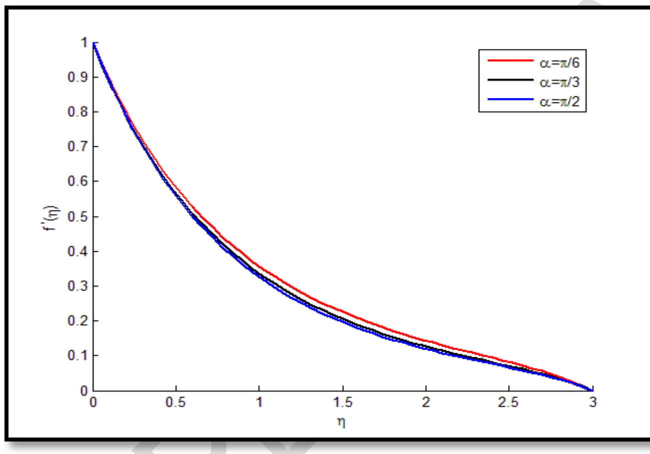


**Figure 4: Velocity profile for different values of angle of inclination**

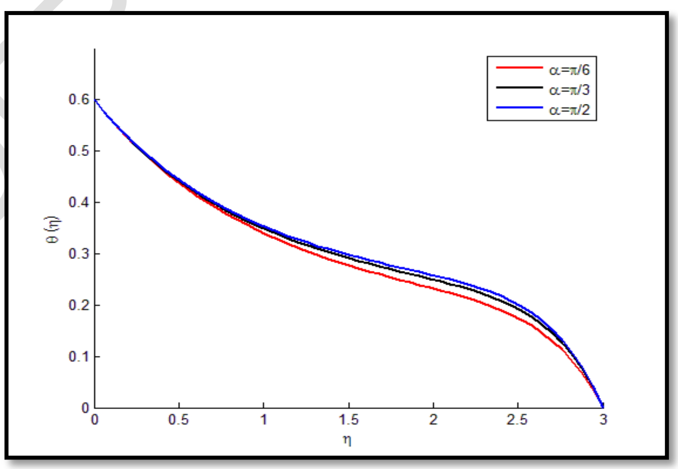


**Figure 5: Temperature profile for different values of angle of inclination**

### 3.3:Effects of stretching parameter on velocity and temperature profiles

*Fig.6* Shows the effect of exponential stretching parameter $N$ to the velocity profile $f'(\eta)$. It is noted that the fluid velocity increases with increase in $N$ .This is because stretching of the sheet wall reduces the momentum boundary layer which leads to the reduction of the viscosity which in turn make the fluid to flow faster.

*Fig.7* depict the effect of exponential parameter $N$ on temperature profile. It is noted that the temperature decreases with increasing $N$ due to the fact that the thermal boundary layer thickness decreases with increasing $N$ .This makes the wall temperature to decrease throughout the boundary layer.

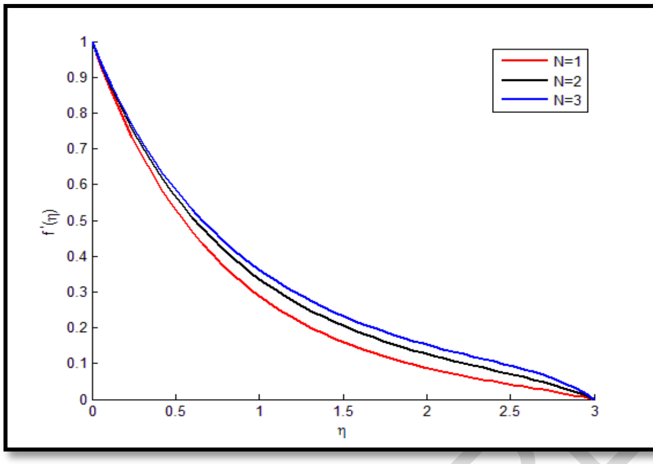


**Figure 6: Velocity profile for different values of exponential stretching parameter**

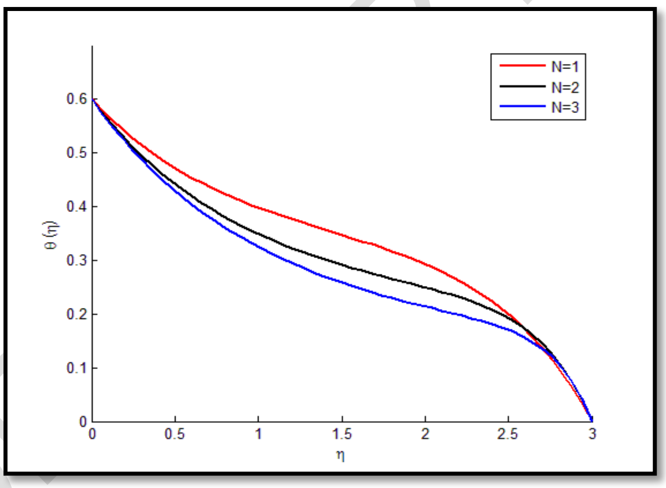


**Figure 7: Temperature profile for different values of exponential stretching parameter**

**3.4:Effects of stratification parameter on velocity and temperature profiles**

Fig.8 shows temperature profile $\theta'(\eta)$ for various values of stratified parameter. It is noted that the temperature decreases as the stratified parameter increases. This is due to the fact that increase of stratification parameter $St$ means decrease in surface temperature. This makes the thermal boundary layer thickness to decrease leading to less heat diffusion thus decreasing the temperature profile. On velocity profile, variation in stratification parameter has no observable effect.

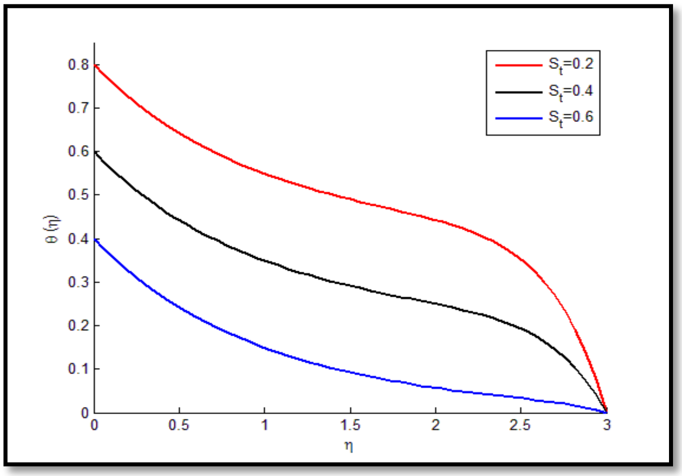


**Figure 8: Temperature profile for different values of stratification parameter**

### 3.5: Effects of Prandtl number on velocity and temperature profiles

Prandtl number $\Pr$ is the ratio of momentum diffusivity to thermal diffusivity. In heat transfer, it controls the relative thickness of the momentum and thermal boundary layer. In *fig.9*, the temperature decreases with increase of Prandtl number for some length $\eta < 2.25$ .This is because increase in Prandtl number makes thermal boundary layer to decrease. This makes heat diffusion to be slow and therefore thermal conductivity becomes small resulting to a decrease in temperature profile. Beyond $\eta = 2.25$ there is an insignificant rise in temperature. For velocity profile, Prandtl number has no effect.

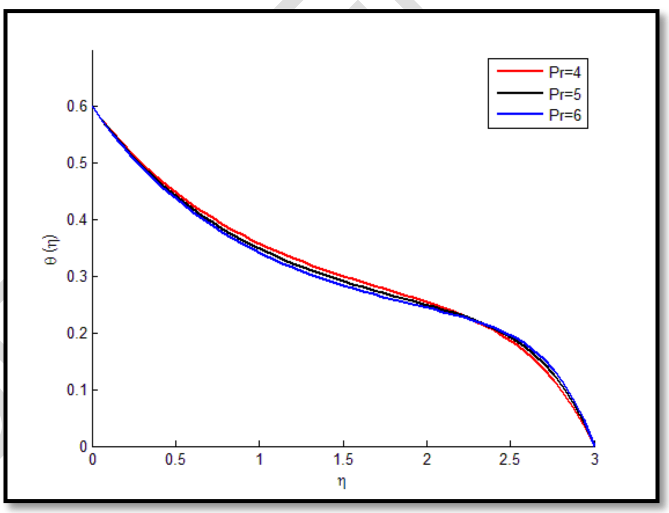


**Figure 9: Temperature profile for different values of Prandtl number**

### 3.6: Effects of Radiative parameter on velocity and temperature profiles

Fig.10 depict the effect of radiation parameter $Rd$ on temperature profile. It is noted that as the $Rd$ increases the temperature increases. This is due to enhancement of thermal boundary layer thickness which provides more heat to the fluid and this result to an enhancement in the temperature profile. Beyond $\eta = 2.25$ higher temperature is not maintained, therefore it drops drastically. No effect of radiation on the velocity profile.

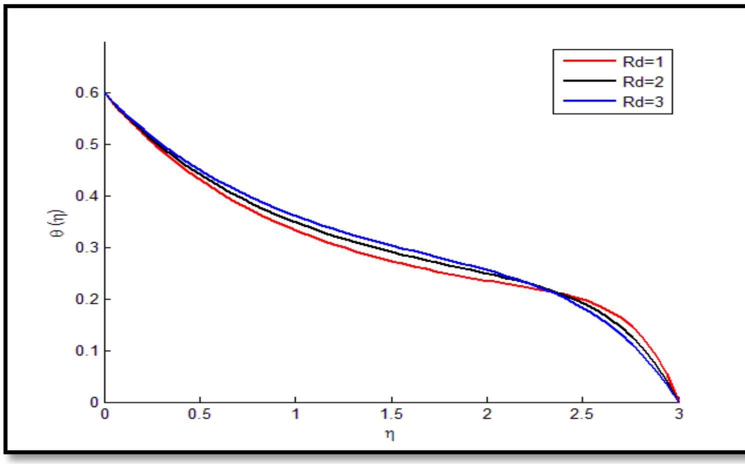


**Figure 10: Temperature profile for different values of Radiative profile**

### 3.7: Effects of parameters variation on the skin friction $C_f$ and Nusselt number $Nu$ .

Table 1 shows the effect of magnetic $M$ , angle of inclination $\alpha$ , permeability $k_1$ , radiation $Rd$ , injection $S$ , Prandtl number $\Pr$ and the exponential stretching $N$ parameters on skin friction coefficient $C_f$ and Nusselt number $Nu$ . It is noted that the skin friction and Nusselt number are associated with fluid velocity and heat transfer rate respectively. From the table it is observed that increase in permeability parameter $k_1$ results to a reduction in skin friction coefficient and decrease in Nusselt number. Permeability being the measure of the ability of a porous material to allow the fluid to pass through it, the increase in this parameter reduces the shear stress, which is the measure of the force of friction. This reduced shear stress results to a reduction in skin friction. For Nusselt number, permeability results to a decrease inviscosity.Now that the fluid injection is constant,the overall fluid temperature will increase to a maximum point thus resulting to less heat transfer.

We also observe that exponential stretching parameter results to an increase in skin friction and increase in Nusselt number. This is attributed to the fact that stretching of the wall surface results to a decrease in velocity boundary layer thus increase in fluid velocity. This in turn increases the contact of fluid particles with the surface, thus increasing the skin friction. For Nusselt number, increase in stretching parameter reduces the momentum boundary layer which makes more fluid particles to be in contact with the surface of the sheet. This enhances heat transfer in the fluid through convection, thus increasing the Nusselt number.

Injection (blowing) through the wall results to an increase in skin friction and decrease in Nusselt number. Increase in Skin friction is due to the pushing of the heated fluid away from the wall, resulting to less viscosity on the wall. For Nusselt number, injection increases the overall fluid temperature to a maximum point. This reduces the heat transfer rate, hence decrease in Nusselt number

When strength of magnetic field and angle of inclination increases, it is noted that Skin friction coefficient decreases and Nusselt number insignificantly decreases. The physical explanation given is that increase in magnetic field strength and angle of inclination makes the Lorentz force large thus reducing the fluid motion. This increases the no slip effect thus decreasing the skin friction. For Nusselt number, the decrease is due to less transfer of the heat from one point to another in the flow field as a result of increased overall fluid temperature.

Parameters such as $St$ , $Rd$ and $\Pr$ have no effect on Skin friction coefficient even if they are varied. Also on Nusselt number, $St$ has no effect. But increase in $Rd$ and $\Pr$ makes Nusselt number decrease and increase respectively. This is due to the fact that higher $\Pr$ fluid has relatively lower thermal conductivity which reduces conduction and thereby increasing heat transfer rate at the surface. Larger values of $Rd$ results to an increase in thickness of thermal boundary layer which results to a decrease on heat transfer.

**Table 1: Values of Skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ for various parameters**

| $N$ | $St$ | $\alpha$ | $M$ | $Rd$ | Pr | $f''(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.4 | $\pi/3$ | 0.4 | 2 | 5 | -0.9699 | 0.3975 |
| 2 | | | | | | -1.2321 | 0.3728 |
| 1 | | | | | | -1.2066 | 0.3174 |
| 2 | | | | | | -1.1051 | 0.3846 |
| 3 | | | | | | -1.0548 | 0.4119 |
| 2 | | | | | | -1.4839 | 0.5797 |
| | 0.4 | | | | | -0.8616 | 0.2846 |
| | 0.2 | | | | | -1.1051 | 0.3846 |
| | 0.4 | | | | | -1.1051 | 0.3846 |
| | 0.6 | $\pi/3$ | | | | -1.1051 | 0.3846 |
| | 0.4 | $\pi/6$ | | | | -1.0520 | 0.3897 |
| | | $\pi/3$ | | | | -1.1051 | 0.3846 |
| | | $\pi/2$ | 0.4 | | | -1.1311 | 0.3822 |
| | | $\pi/3$ | 0.2 | | | -1.0654 | 0.3884 |
| | | | 0.4 | | | -1.1051 | 0.3846 |
| | | | 0.6 | 2 | | -1.1440 | 0.3810 |
| | | | 0.4 | 1 | | -1.1051 | 0.4100 |
| | | | | 2 | | -1.1051 | 0.3846 |
| | | | | 3 | 5 | -1.1051 | 0.3627 |
| | | | | 2 | 4 | -1.1051 | 0.3692 |
| | | | | | 5 | -1.1051 | 0.3846 |
| | | | | | 6 | -1.1051 | 0.3958 |

## 4. Conclusion

The boundary layer flow of a viscous incompressible fluid over exponential stretching sheet with an inclined magnetic field in presence of thermal radiation was studied. The governing continuity, momentum and energy equations were obtained through similarity transformations. The higher order non-linear differential equations obtained were reduced to a system of first order ordinary differential equations. The solutions were computed numerically by collocation method. The numerical results for the governing parameters were presented graphically. Also various numerical values for skin friction and Nusselt number were obtained for each parameter. Some of the main conclusions made are:

i. Velocity profile decreases with increase in strength of magnetic field and angle of inclination
ii. There is increase in temperature profile when the strength of magnetic field, angle of inclination, of the material increases. It also increases when the radiative Property of the material increases.
iii. Temperature profile decreases with increase in exponential stretching of the material, stratification of the material and Prandtl number.
iv. There is decrease in skin friction when angle of inclination increases but increases with increase in exponential stretching of the material.

v. Nusselt number increases with increase in exponential stretching, and Prandtl number but decreases with increase in magnetic field, angle of inclination and radiative property of the material.

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