Original Research Article

On the Global Stability Analysis of Corona Virus Disease (COVID-19) Mathematical Model

ABSTRACT

In this present work, we investigated the Global Stability Analysis of Corona virus disease model formulated by Atokolo et al in [11].The COVID‑19 pandemic, also known as the coronavirus pandemic, is an ongoing pandemic that is ravaging the whole world. By constructing a Lyapunov function, we investigated the stability of the model Endemic Equilibrium state to be globally asymptotically stable. This results epidemiologically implies that the COVID-19 will invade the population in respective of the initial conditions (population) considered.

Keywords: corona; virus;disease;global;stability

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1.0 Introduction

Corona virus popularly known as (COVID-19) is an ongoing pandemic disease caused by severe acute respiratory syndrome Coronavirus 2, [1,2]. The disease was first discovered in Wuhan, China in December 2019 [2] and was declared to be a public health emergency of international concern on the 30th January, 2020 and was identified as a global pandemic by the World Health Organization(WHO) on the 11th March, 2020 [3,4]. 1.94 million Cases were reported as of 14th April, 2020 in over 210 countries of the world, leading to over 121,000 deaths with at least 465,000 recoveries including Nigeria,[5, 6,7].

Fever, cough, shortness of breath, pneumonia and acute respiratory distress syndrome were identified as possible symptoms of corona virus,[8,9].The disease has an incubation period of 2-14 days[10]. The virus majorly is contracted during close contact and also by small droplets produced in the process of sneezing, talking and coughing,[2].In this present work, we extend the work of Atokolo et al in [11] by conducting a Global Stability Analysis(GAS) on the corona virus model.

The concept of global stability is concerned with global properties of a model which can be investigated using Lyapunov function theory. Lyapunov gave a technique that can show if an equilibrium state is stable or unstable through the construction of a Lyapunov function. Lyapunov functions are positive functions that reduce in time along the orbits of a model. The method is advantageous because it sometimes proves stability of a non hyperbolic equilibrium,[12], however, there is no direct method of constructing a Lyapunov functions.

An equilibrium state is asymptotically stable globally if its property holds globally and its domain of attraction is the entire space,[12]. Models such as the ones in [12, 13, 14] are veritable tools towards studying global stabilities of biological models.

2.0 Model Formulation and Procedures:

The following assumptions were provided by Atokolo et al in [11].In modelling the spread of disease (COVID'19) pandemic, the following were assumed.

i. The model incorporates a net inflow of individuals into the susceptible population. This parameter comprises of new births, immigration and emigration.
ii. All classes of the population die naturally.
iii. Disease induced death is considered in the model.
iv. Infected individuals can recover naturally though the rate is assumed to be minimal.
v. The Recovered has permanent immunity for re-infection.
vi. Every individual taken for treatment recover at a high rate, that is to say the treatment is considered to be effective.
vii. The population is divided into the Susceptible class (S), the Exposed class (E), Quarantine class (Q), Isolated class (J), the Infected class (I), the Infected but treated class (I_T), and the Recovered class(R).

2.3 Mathematical Model for the transmission and control of COVID-19

The mathematical model that incorporates the above assumptions as given in [11] is given as:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda - \alpha(1-x)S - \mu S \\
\frac{dE}{dt} &= \alpha(1-x)S - [\theta(1+y) + \beta + \mu]E \\
\frac{dQ}{dt} &= \theta(1+y)E - (\eta + \mu)Q \\
\frac{dJ}{dt} &= \eta Q + \emptyset(1+z)I - (\mu + \sigma + r + \rho)J \\
\frac{dI}{dt} &= \beta E - [\emptyset(1+Z) + \lambda + \sigma + \mu]I \\
\frac{dI_T}{dt} &= \gamma J - (\omega + \mu + \sigma)I_T \\
\frac{dR}{dt} &= \lambda I + \rho J + \omega I_T - \mu R
\end{aligned}\right\} \quad (1)$$

Where $\alpha = \frac{\alpha_1 E + \alpha_2 Q + \alpha_3 J + \alpha_4 I + \alpha_S I_T}{N}$

and $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$

where $x, y, z,$ are control parameters

2.4 Model Variables and Parameters Description

Table 1: Model Variables and Description

S/N	VARIABLES	DESCRIPTION
1.	S	Susceptible Human
2.	E	Exposed Human `
3.	Q	Quarantined Human
4.	J	Isolated Human
5.	I	Infected Human
6.	I_T	Infected but treated Human
7.	R	Recovered Human

Table 2: Model parameters and Description

S/N	VARIABLES	DESCRIPTION
1	$\wedge$	Recruitment rate
2	α	Force of infection
3	θ	Rate at which the exposed are quarantine
4	η	Rate at which the quarantined are isolated
5	β	Rate at which the exposed are infected
6	$\emptyset$	Rate at which the infected are isolated
7	γ	Treatment rate
8	ρ	Natural recovery rate of the isolated
9	λ	Natural recovery rate of the infected
10	ω	Recovery rate due to treatment
11	σ	Disease induced death rate
12	μ	Natural death rate
13	x	Enlightenment control measures for the susceptible individuals to observe social distance, washing of hands, covering of mouth when talking, coughing and sneezing
14	y	Enlightenment control measure for the exposed to be quarantined
15	Z	Enlightenment control measure for the infected to be isolated

(2.5) Endemic Disease Equilibrium (EE)

At the endemic disease equilibrium, infection exists and as such we let:

$$S = S^*, E = E^*, Q = Q^*, J = J^*, I = I^*, I_T = {I_T}^* \text{ and } R = R^*$$

Also at equilibrium,

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dQ}{dt} = \frac{dJ}{dt} = \frac{dI}{dt} = \frac{d\boldsymbol{I_T}}{dt} = \frac{dR}{dt} = 0$$

The EE is represented by $[S^*, E^*, Q^*, J^*, I^*, {I_T}^* \; R^*]$ which is given by:

$$\varepsilon_1 = \left\{ \begin{matrix} \frac{\alpha(1-x)}{[\theta(1+y)+\beta+\mu]E^*}, \frac{\beta}{[\emptyset(1+z)+\lambda+\sigma+\mu]}, \frac{\theta(1+y)E^*}{(\eta+\mu)}, \frac{(\omega+\mu+\sigma)I_T{}^*}{\gamma}, \\ \frac{(\eta+\mu)(\mu+\sigma+\gamma+\rho)(\omega+\mu+\sigma)\, I_T{}^* - \gamma\eta\theta(1+y)E^*}{\emptyset\gamma(\eta+\mu)(1+z)}, \frac{(\lambda+\mu\gamma\lambda)I^*}{[\omega\gamma\mu+\rho\mu\omega(\omega+\mu+\sigma)]}, \frac{\gamma\lambda I^*+[\rho(\omega+\mu+\sigma)+\gamma\omega]I_T{}^*}{\gamma\mu} \end{matrix} \right\} \quad (2)$$

3.0 Global Stability Analysis Of Endemic Equilibrium Point of The Model

Theorem 2: If $R_0 > 1$, the endemic equilibrium point (ε_1) of the model(1) is globally asymptotically stable.

__**Proof:**__

To establish the global stability of the endemic equilibrium point ε_1, we construct the following using Lyapunov function.

$V(S^*\, E^*\, Q^*\, J^*\, I^*\, I_T^*\, R^*) = \left(S - S^* - S^* log\frac{S^*}{S}\right) + \left(E - E^* - E^* log\frac{E^*}{E}\right) + \left(Q - Q^* -\right.$

$Q{*}logQ{*}Q{+}J{-}J{*}{-}J{*}logJ{*}J{+}I{-}I{*}{-}I{*}logI{*}I{+}IT{-}IT{*}{-}IT{*}logIT{*}IT{+}R{-}R{*}{-}R{*}logR{*}R$

(3)

Where:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \wedge - \left(\frac{\alpha_1 E+\alpha_2 Q+\alpha_3 J+\alpha_4 I+\alpha_5 I_T}{N}\right)(1-x)S - \mu S \\
\frac{dE}{dt} &= \left(\frac{\alpha_1 E+\alpha_2 Q+\alpha_3 J+\alpha_4 I+\alpha_5 I_T}{N}\right)(1-x)S - [\theta(1+y)+\beta+\mu]E \\
\frac{dQ}{dt} &= \theta(1+y)E - (\eta+\mu)Q \\
\frac{dJ}{dt} &= \eta Q + \emptyset(1+Z)I - (\mu+\sigma+r+\rho)J \\
\frac{dI}{dt} &= \beta E - [\emptyset(1+Z)+\lambda+\sigma+\mu]I \\
\frac{dI_T}{dt} &= \gamma J - \ (\omega+\mu+\sigma)I_T \\
\frac{dR}{dt} &= \ \lambda I + \rho J + \omega I_T - \mu R
\end{aligned}\right\} \quad (4)$$

Thus:

$$\frac{dV}{dt}=\frac{S-S^*}{S}\left[\wedge-\left(\frac{\alpha_1E+\alpha_2Q+\alpha_3J+\alpha_4I+\alpha_5I_T}{N}\right)(1-x)S-\mu S\right]+\ \frac{E-E^*}{E}\left[\left(\frac{\alpha_1E+\alpha_2Q+\alpha_3J+\alpha_4I+\alpha_5I_T}{N}\right)(1-x)S-\right.$$

θ1+y+β+μE+Q−Q∗Qθ1+yE−η+μQ+J−J∗JηQ+Ø1+ZI−μ+σ+r+ρJ+I−I∗IβE−[Ø(1+Z)+λ+σ+μ]I +IT−IT∗ITγJ− (ω+μ+σ)IT+R−R∗RλI+ρJ+ωIT−μR (5)

Therefore:-

$$\frac{dV}{dt}=(S-S^*)\,\frac{\left[\wedge-\frac{1}{N}(\alpha_1(E-E^*)+\alpha_2(Q-Q^*)+\alpha_3(J-J^*)+\alpha_4(I-I^*)+\alpha_5(I_T-I_T^*))(1-x)(S-S^*)-\mu(S-S^*)\right]}{S}+$$

$$(E-E^*)\left[\frac{\frac{1}{N}(\alpha_1(E-E^*)+\alpha_2(Q-Q^*)+\alpha_3(J-J^*)+\alpha_4(I-I^*)+\alpha_5(I_T-I_T^*))(1-x)(S-S^*)-[\theta(1+y)+\beta+\mu](E-E^*)}{E}\right]+$$

$$\frac{(Q-Q^*)[\theta(1+y)(E-E^*)-(\eta+\mu)(Q-Q^*)]}{Q}+\frac{(J-J^*)[\eta(Q-Q^*)+\emptyset(1+Z)(J-J^*)-(\mu+\sigma+r+\rho)(J-J^*)]}{J}+$$

$$\frac{(I-I^*)[\beta(E-E^*)-[\emptyset(1+Z)+\lambda+\sigma+\mu](I-I^*)]}{I}+\frac{(I_T-I_T^*)[\gamma(J-J^*)-\ (\omega+\mu+\sigma)(I_T-I_T^*)]}{I_T}+$$

$$\frac{(R-R^*)[\lambda(I-I^*)+\rho(J-J^*)+\omega(I_T-I_T^*)-\mu(R-R^*)]}{R}\qquad(6)$$

Then we have :

$$\frac{dV}{dt}=\frac{1}{S}\left[\left[(S-S^*)\wedge-\frac{\alpha_1}{N}(E-E^*)(S-S^*)^2+\frac{\alpha_2}{N}(Q-Q^*)(S-S^*)^2+\frac{\alpha_3}{N}(J-J^*)(S-S^*)^2+\right.\right.$$

α4N(I−I∗)S−S∗2+α5N(IT−IT∗)S−S∗21−x−μS−S∗2+1Eα1N(E−E∗)2S−S∗+α2N(E−E∗)(Q −Q∗)S−S∗+α3N(E−E∗)(J−J∗)S−S∗+α4N(E−E∗)(I−I∗)S−S∗+α5N(E−E∗)(IT−IT∗)S−S∗1− x−θ1+y+β+μ(E−E∗)+1Qθ(Q−Q∗)1+y(E−E∗)−η+μ(Q−Q∗)2+1Jη(J−J∗)(Q−Q∗)+Ø1+Z(I −I∗)(J−J∗)−μ+σ+r+ρ(J−J∗)2+1Iβ(I−I∗)(E−E∗)−[Ø(1+Z)+λ+σ+μ](I−I∗)2+1ITγ(IT−IT∗)(J−J∗)− (ω+μ+σ)(IT−IT∗)2+1Rλ(R−R∗)(I−I∗)+ρ(J−J∗)+ω(IT−IT∗)−μ(R−R∗)2 (7)

Therefore:

$$\frac{dV}{dt}=\frac{\wedge(1-x)(S-S^*)}{S}-\frac{\alpha_1(1-x)}{N}\frac{(E-E^*)(S-S^*)^2}{S}+\frac{\alpha_2(1-x)}{N}\frac{(Q-Q^*)(S-S^*)^2}{S}+\frac{\alpha_3(1-x)}{N}\frac{(J-J^*)(S-S^*)^2}{S}+$$

$$\frac{\alpha_4(1-x)}{N}\frac{(I-I^*)(S-S^*)^2}{S}+\frac{\alpha_5(1-x)}{N}\frac{(I_T-I_T^*)(S-S^*)^2}{S}-\frac{\mu(S-S^*)^2}{S}+\frac{\alpha_1(1-x)}{N}\frac{(E-E)^*(S-S^*)}{S}+$$

$$\frac{\alpha_2(1-x)}{N}\frac{(E-E^*)(Q-Q^*)(S-S^*)}{S}+\frac{\alpha_3(1-x)}{N}\frac{(E-E^*)(J-J^*)(S-S^*)}{S}+\frac{\alpha_4(1-x)}{N}\frac{(E-E^*)(I-I^*)(S-S^*)}{S}+$$

$$\frac{\alpha_5(1-x)}{N}\frac{(E-E^*)(I_T-I_T^*)(S-S^*)}{S}-\frac{[\theta(1+y)+\beta+\mu](E-E^*)}{E}+\frac{(Q-Q^*)\theta[(1+y)(E-E^*)]}{Q}-\frac{(\eta+\mu)(Q-Q^*)^2}{Q}+$$

$$\frac{(J-J^*)\eta(Q-Q^*)}{J}+\frac{(J-J^*)\emptyset(1+Z)(I-I^*)}{J}-\frac{(\mu+\sigma+r+\rho)(J-J^*)^2}{J}+\frac{(I-I^*)\beta(E-E^*)}{I}-\frac{(I-I^*)^2[\emptyset(1+Z)+\lambda+\sigma+\mu]}{I}+$$

$$\frac{(I_T-I_T^*)\gamma(J-J^*)}{I_T}-\frac{(\omega+\mu+\sigma)(I_T-I_T^*)^2}{I_T}+\frac{(R-R^*)\lambda(I-I^*)}{R}+\frac{(R-R^*)\rho(J-J^*)}{R}+\frac{(R-R^*)\omega(I_T-I_T^*)}{R}-\frac{\mu(R-R^*)^2}{R} \quad (8)$$

Collecting the positive and negative terms from equation (8)

We obtain $\frac{dV}{dt}=M_1-M_2$

Where M_1 represents the positive terms and M_2 represents the negative terms in the expression (8)

$$M_1=\frac{(S-S^*)^2}{S}\left[\frac{\alpha_2(1-x)(Q-Q^*)}{N}+\frac{\alpha_3(1-x)(J-J^*)}{N}+\frac{\alpha_4(1-x)(I-I^*)}{N}+\frac{\alpha_5(1-x)(I_T-I_T^*)}{N}\right]+\frac{(E-E^*)^2}{E}\frac{\alpha_1}{N}(1-x)(S-S^*)+$$
$$\frac{(S-S^*)}{S}\Lambda(1-x)+\frac{(E-E^*)}{E}\left[\frac{\alpha_2(1-x)(Q-Q^*)(S-S^*)}{N}+\frac{\alpha_3(1-x)(J-J^*)(S-S^*)}{N}+\frac{\alpha_4(1-x)(I-I^*)(S-S^*)}{N}+\frac{\alpha_5(1-x)(I_T-I_T^*)(S-S^*)}{N}\right]+$$
$$\frac{(Q-Q^*)}{Q}\theta(1+y)(E-E^*)+\frac{(J-J^*)}{J}[\eta(Q-Q^*)+\emptyset(1+Z)(I-I^*)]+\frac{(I-I^*)}{I}\beta(E-E^*)+\frac{(I_T-I_T^*)}{I_T}\gamma(J-J^*)+$$
$$\frac{(R-R^*)}{R}[\gamma(I-I^*)+\rho(J-J^*)+\omega(I_T-I_T^*)] \quad (9)$$

and

$$M_2=\frac{(S-S^*)}{S}\left[\frac{\alpha_1}{N}(1-x)(E-E^*)+\mu\right]+\frac{(Q-Q^*)^2}{Q}(\eta+\mu)+\frac{(J-J^*)^2}{J}(\mu+\sigma+\gamma+\rho)+\frac{(I-I^*)^2}{I}(\emptyset(1+$$

$Z+\lambda+\sigma+\mu+(IT-IT*)2IT\omega+\mu+\sigma+(R-R*)R\mu+E-E*E\theta1+y+\beta+\mu \qquad (10)$

Therefore, if $M_1<M_2$, then $\frac{dV}{dt}$ will be negative definite along the solution path of the system. Thus this implies that $\frac{dV}{dt}<0$, and $\frac{dV}{dt}=0$ only at a point where $S=S^*, E=E^*, Q=Q^*, J=J^*, I=I^*, I_T=I_T^*\ R=R^*$.

Therefore, the largest compact set is $\left\{(S^*,E^*,Q^*,J^*,I^*,I_T^*\ R^*)\in\Omega\frac{dV}{dt}=0\right\}$ is just the singleton set (ε_1) where (ε_1) is the endemic equilibrium point.

According to Lasalle's Invariant Principle in [15],it therefore means that (ε_1) is globally assymptotically stable in Ω if $M_1<M_2$.

This results epidemiologically implies that the COVID-19 will invade the population in respective of the initial conditions (population) considered.

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