Original Research Article

MEASURE OF MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING BALANCED INCOMPLETE BLOCK DESIGNS

ABSTRACT

In this paper, measure of modified rotatability for second order response surface designs using BIBD is suggested which enables us to assess the degree of modified rotatability for a given response surface design.

Keywords and phrases: Response surface designs, modified rotatable designs, balanced incomplete block designs, measure of rotatability for second order response surface designs.

1. Introduction

Response surface methodology is a collection of mathematical and statistical techniques useful for analysing problems where several independent variables influence a dependent variable. The independent variables are often called the input or explanatory variables and the dependent variable is often the response variable. An important step in development of response surface designs was the introduction of rotatable designs by Box and Hunter (1957). Das and Narasimham (1962) constructed rotatable designs using balanced incomplete block designs (BIBD). A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design centre. Das et al. (1999) introduced modified second order response surface designs. Park et al. (1993) introduced measure of rotatability for second order response surface designs. Victorbabu and Vasundharadevi (2005) suggested modified second order response surface designs using BIBD. Victorbabu et al. (2006) suggested modified second order response surface designs, rotatable designs using pairwise balanced design. Victorbabu et al. (2008) suggested modified rotatable central composite designs. Victorbabu and Vasundharadevi (2008) studied modified second order response surface designs, rotatable designs using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Victorbabu and Surekha (2012) developed measure of rotatability for second order response surface designs using central composite designs. Victorbabu and Surekha (2013) suggested measure of rotatability for second order response surface designs using incomplete block designs. Victorbabu and Surekha (2015) studied measure of rotatability for second order response surface designs using BIBD. Victorbabu and Jyostna (2017) studied measure of rotatability for second order response surface designs using a pair of BIBD. Victorbabu and Jyostna (2020) studied measure of modified rotatability for second order response surface designs. These measures are useful to enable us to assess the degree of modified rotatability for a given second order response surface designs.

2. Conditions for second order rotatable designs

Suppose we want to use the second order response surface design D= $((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + e_u \qquad (1)$$

where x_{iu} denotes the level of the i^{th} factor (i =1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2 is said to be second order rotatable design (SORD), if the variance of the estimate of first order partial derivative of $Y_u(x_1, x_2, ..., x_v)$ with respect to each of independent variables (x_i) is only a function of the distance ($d^2 = \sum_{i=1}^{v} x_i^2$) of the point (x_1,x_2, …,x_v) from the origin (centre) of the design. Such a spherical variance function for estimation of second order response surface is achieved if the design points satisfy the following conditions [cf. Box and Hunter (1957), Das and Narasimham (1962)].

1. $\sum x_{iu} = 0$, $\sum x_{iu} x_{ju} = 0$, $\sum x_{iu} x_{ju}^2 = 0$, $\sum x_{iu} x_{ju} x_{ku} = 0$, $\sum x_{iu}^3 = 0$, $\sum x_{iu} x_{ju}^3 = 0$, $\sum x_{iu} x_{ju} x_{ku}^2 = 0$, $\sum x_{iu} x_{ju} x_{ku} x_{lu} = 0$; for $i \neq j \neq k \neq l$; (2)
2. (i) $\sum x_{iu}^2 =$ constant $= N\lambda_2$;

   (ii) $\sum x_{iu}^4 =$ constant $= cN\lambda_4$; for all i (3)
3. $\sum x_{iu}^2 x_{ju}^2 =$ constant $= N\lambda_4$; for $i \neq j$ (4)
4. $\sum x_{iu}^4 = c \sum x_{iu}^2 x_{ju}^2$ (5)
5. $\dfrac{\lambda_4}{\lambda_2^2} > \dfrac{v}{(c+v-1)}$ (6)

where c, λ_2 and λ_4 are constants and the summation is over the design points.

If the above mentioned conditions are satisfied, the variances and covariances of the estimated parameters become,

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]} \ ,$$

$$V(\hat{b}_i)=\frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij})=\frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii})=\frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii})=\frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \qquad (7)$$

and other covariances are zero.

3. Conditions for modified second order rotatable designs

The most widely used design for fitting a second order model is the central composite design. Central composite designs are constructed by adding suitable factorial combinations to those obtained from $\frac{1}{2^p}\times 2^v$ fractional factorial design (here $2^{t(v)}=\frac{1}{2^p}\times 2^v$ denotes a suitable fractional replicate of 2^v, in which no interaction with less than five factors is confounded). In coded form the points of $2^v(2^{t(v)})$ factorial have coordinates $(\pm a,\pm a,...,\pm a)$ and $2v$ axial points have coordinates of the form ($(\pm b, 0,...0), (0, \pm b,...0), . . . ,(0, 0, ..., \pm b)$) etc., and n_0 central points. The usual method of construction of SORD is to take combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant. All such combinations form a design. Generally, SORD need at least five levels (suitably coded) at $0, \pm a, \pm b$ for all factors ($(0,0,...0)$)- chosen centre of the design, unknown level 'a' and 'b' are to be chosen suitably to satisfy the conditions of the rotatability) generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively, by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SORD the restriction used is $\sum x_{iu}^4 = 3\sum x_{iu}^2 x_{ju}^2$, i.e., $c=3$. Other restrictions are also possible through, it seems, not exploited well. Das et al (1999) proposed the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ i.e., $\lambda_2^2 = \lambda_4$ to get another series of symmetrical second order response surface designs, which provide more precise estimates of response at specific points of interest than what is available from the corresponding existing designs. Further, the variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\sigma^2}{N\sqrt{\lambda_4}(c-1)} \qquad (8)$$

and other covariances are zero. These modifications of the variances and covariances affect the variance of the estimated response at specific points considerably. Using these variances and covariances, variance of estimated response at any point can be obtained. Let $\hat{y}_u$ denote the estimated response at the point $(x_{1u}, x_{2u}, \ldots x_{vu})$. Then,

$$V(\hat{Y}_u) = V(\hat{b}_0) + d^2[V(\hat{b}_i) + 2cov(\hat{b}_0, \hat{b}_{ii})] + d^4 V(\hat{b}_{ii}) + (\sum x_{iu}^2 x_{ju}^2)[(c-3)\sigma^2 / (c-1)N\lambda_4]$$

Construction of modified response surface designs is the same as for SORD except that instead of taking $c=3$ the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ is to be used and this condition will provide different values of the unknowns involved. (cf. Das et al. 1999).

4. Conditions for measure of rotatability for second order response surface designs

Following Box and Hunter (1957), Das and Narasimham (1962), Park et al (1993), conditions (2) to (6) and (7) give the necessary and sufficient conditions for a measure of rotatability for any general second order response surface designs. Further we have,

$V(b_i)$ are equal for i,

$V(b_{ii})$ are equal for i,

$V(b_{ij})$ are equal for i, j, where $i \neq j$,

$$Cov(b_i, b_{ii}) = Cov(b_i, b_{ij}) = Cov(b_{ii}, b_{ij}) = Cov(b_{ij}, b_{il}) = 0 \text{ for all } i \neq j, j \neq l, l \neq i. \quad (9)$$

Park et al. (1993) suggested that if the conditions in (2) to (6) together with (7) and (9) are met, then the following measure $(P_v(D))$ given below can be used to assess the degree of rotatability for any general second order response surface design (cf. Park et al., 1993).

$$P_v(D) = \frac{1}{1+R_v(D)}, \quad (10)$$

where

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \frac{6v\left[V(\hat{b}_{ij}) + 2\,cov(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})\right]^2 (v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \quad (11)$$

and g is the scaling factor.

On simplification, numerator of (11), $[V(\hat{b}_{ij}) + 2\,cov(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})]$ using (7) becomes $(c-3)\sigma^2 / (c-1)N\lambda_4$. Thus $R_v(D)$ becomes

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v[(c-3)\sigma^2]^2(v-1)}{[(c-1)N\lambda_4]^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right) \quad (12)$$

Note: For SORD, we have $c = 3$. Substituting the value of $'c'$ in (12) and on simplification we get $R_v(D)$ is zero. Hence from (10), we get $P_v(D)$ is one if and only if a design is rotatable and less than one then it is nearly rotatable design.

5. Modified rotatability for second order response surface designs using BIBD

Balanced Incomplete Block Design: A BIBD denoted by (v,b,r,k,λ) is an arrangement of v treatments in b blocks each containing $k(< v)$ treatments, if (i) every treatment occurs at most once in a block, (ii) every treatment occurs in exactly r blocks and (iii) every pair of treatments occurs together in λ blocks.

The result of modified rotatability for second order response surface designs using BIBD is suggested here (cf. Victorbabu and Vasundharadevi 2005). Let (v,b,r,k,λ) be a BIBD, $2^{t(k)}$ denotes a fractional replicate of 2^k with $+1$ or -1 levels in which no interaction with less than five factors is confounded. $[1-(v,b,r,k,\lambda)]$ denote the design points generated from the transpose of the incidence matrix of BIBD. $[1-(v,b,r,k,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from BIBD by "multiplication" (cf. Raghavarao, 1971). Repeat $b2^{t(k)}$ design points y_1 times. Let $(\pm a,0,0,\ldots 0)2^1$ denote the design points generated from $(\pm a,0,0,\ldots 0)$ point set. Repeat this set of additional design points, say y_2 times. Let n_0 be the number of central points in modified SORD and U denotes combination of the design points generated from different sets of points, when $r < 3\lambda$.

The design points, $y_1[1-(v,b,r,k,\lambda)]2^{t(k)}\ U\ y_2(\pm a,0,0,\ldots.0)2^1\ U\ (n_0)$ will give a v-dimensional modified SORD in $N = \dfrac{(y_1 r 2^{t(k)} + 2y_2 a^2)^2}{y_1\lambda 2^{t(k)}}$ design points if,

$$a^4 = \frac{(3\lambda-r)y_1 2^{t(k)-1}}{y_2},$$

$$n_0 = \frac{(y_1 r 2^{t(k)} + 2y_2 a^2)^2}{y_1\lambda 2^{t(k)}} - [y_1 b 2^{t(k)} + 2y_2 v]$$ and n_0 turns out to be an integer.

6. Measure of rotatability for second order response surface designs using BIBD

The result of measure of rotatability for second order response surface designs using BIBD is suggested here (cf. Victorbabu and Surekha, 2015). Let (v,b,r,k,λ) denote a BIBD. The design points, $y_1[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}$ U $y_1(a,0,0,....0)2^1$ U (n_0) will give a measure of rotatability for second order response surface designs using BIBD in $N = y_1 b 2^{t(k)} + 2vy_2 + n_0$ design points with level 'a' prefixed and $c = \frac{r2^{t(k)}y_1 + 2y_2a^4}{\lambda 2^{t(k)}y_1}$. (Here we take $y_1 = 1, y_2 = 1$)

We can obtain the measure of rotatability values for second order response surface designs using BIBD. We have

$$R_v(D) = \left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2(v+2)^2(v+4)(v+6)(v+8)g^8}$$

where

$$g = \begin{cases} \dfrac{1}{a}, & \text{if } \; a < \sqrt{\dfrac{2^{t(k)-1}(b\text{-}r)y_1}{y_2} + v} \\[2ex] \dfrac{1}{\sqrt{\dfrac{2^{t(k)-1}(b\text{-}r)y_1}{y_2} + v}}, & \text{if } \; a > \sqrt{\dfrac{2^{t(k)-1}(b\text{-}r)y_1}{y_2} + v} \end{cases}$$

$$P_v(D) = \frac{1}{1+R_v(D)}$$

If $P_v(D)$ is 1 if and only if the design is rotatable, and it is smaller than one for a non-rotatable designs.

7. Measure of modified rotatability for second order response surface designs using BIBD

The proposed measure of modified rotatability for second order response surface designs using BIBD when $r < 3\lambda$ is suggested here. Let (v,b,r,k,λ) denote a BIBD. $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels, such that no interaction with less than five factors is confounded. $[1\text{-}(v,b,r,k,\lambda)]$ denote the design points generated from the transpose of incidence matrix BIBD, $[1-(v,b,r,k,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from BIBD by multiplication. Repeat these $b2^{t(k)}$ design points y_1 times. Let $(\pm a,0,...,0)2^1$ denote the design points generated from $(\pm a,0,...,0)$ point set. Repeat this set of additional design points say y_2 times and n_0 be the number of central points. The method of construction of measure of modified rotatability for second order response surface designs using BIBD is suggested as follows.

The design points,

$y_1[1-(v,b,r,k,\lambda)]2^{t(k)}\ U\ y_2(\pm a,0,0,....0)2^1\ U\ (n_0)$ generated from BIBD, we have,

$$\sum x_{iu}^2 = y_1 r 2^{t(k)} + y_2 2a^2 = N\lambda_2 \qquad (13)$$

$$\sum x_{iu}^4 = y_1 r 2^{t(k)} + y_2 2a^4 = cN\lambda_4 \qquad (14)$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1 \lambda 2^{t(k)} = N\lambda_4 \qquad (15)$$

To make the design rotatable, we take $c = 3$. From equations (14) and (15), we have

$$a^4 = \frac{(3\lambda - r) y_1 2^{t(k)-1}}{y_2},$$

The modified condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ leads to N which is given by

$N = \dfrac{(y_1 r 2^{t(k)} + 2y_2 a^2)^2}{y_1 \lambda 2^{t(k)}}$ alternatively N may be obtained directly as $y_1 b 2^{t(k)} + 2vy_2 + n_0$, where

n_0 is given by $n_0 = \dfrac{(y_1 r 2^{t(k)} + 2a^2 y_2)^2}{y_1 \lambda 2^{t(k)}} - [y_1 b 2^{t(k)} + 2y_2 v]$ and n_0 turns out to be an integer.

From equations (13) and (15) and on simplification we get $\lambda_2 = \dfrac{y_1 r 2^{t(k)} + 2y_2 a^2}{N}$ and $\lambda_4 = \dfrac{y_1 \lambda 2^{t(k)}}{N}$.

To obtain measure of modified rotatability for second order response surface designs using BIBD, we have

$$P_v(D) = \frac{1}{1+R_v(D)}$$

$$R_v(D) = \left[\frac{(c-3)}{(c-1)}\right]^2 \frac{6v(v-1)}{\lambda_4^2(v+2)^2(v+4)(v+6)(v+8)g^8},$$

where g is a scaling factor

$$g = \begin{cases} \frac{1}{b}, & \text{if } b < \sqrt{\frac{2^{t(k)-1}y_1(b-r)}{y_2} + v} \\ \frac{1}{\sqrt{\frac{2^{t(k)-1}y_1(b-r)}{y_2} + v}} & \text{otherwise} \end{cases}$$

The following table gives the values of a measure of modified rotatability for second order response surface designs using BIBD. It can be verified that $P_v(D)$ is 1 if and only if the design is modified rotatable, and it is smaller than one for nearly modified rotatable designs.

Example: We illustrate the measure of modified rotatability for second order response surface designs for $v=5$ factors with the help of a BIBD $(v=5,b=10,r=6,k=3,\lambda=3)$. The design points,

$y_1[1-(v=5,b=10,r=6,k=3,\lambda=3)]2^3 \text{ U } y_2(\pm a,0,0,....0)2^1 \text{ U } (n_0)$ will give a measure of modified rotatability for second order response surface designs in N=150 design points. From (13), (14) and (15), we have

$$\sum x_{iu}^2 = 48+6a^2 = N\lambda_2 \tag{16}$$

$$\sum x_{iu}^4 = 48+6a^4 = cN\lambda_4 \tag{17}$$

$$\sum x_{iu}^2 x_{ju}^2 = 24 = N\lambda_4 \tag{18}$$

Here the measure of modified rotatability for second order response surface designs using BIBD is suggested.

From equations (17) and (18) with rotatability value $c=3$ we get $a^4 = 4 \Rightarrow a^2 = 2 \Rightarrow a=1.414214$ from equations (16) and (18) using the modified condition with $(\lambda_2^2=\lambda_4)$ with $a^2=2$, $y_1=1$ and $y_2=3$, we get $N=150$, $n_0=40$. Here we get for modified SORD $P_v(D)=1$ by taking $a=1.414214$ and scaling factor $g=0.7071$. Then the design is modified SORD using BIBD.

Instead of taking $a=1.414214$ if we take $a=2.2$ for the above BIBD $(v=5,b=10,r=6,k=3,\lambda=3)$ from equations (17) and (18), we get $c=7.8564$. The scaling factor $g=0.5578$, $R_v(D)=2.2199$ and $P_v(D)=0.3106$. Here $P_v(D)$ becomes smaller it deviates from modified rotatability.

The measure of modified rotatability for second order response surface designs using BIBD, at different values of 'a' for $3 \le v \le 11$. It can be verified that $P_v(D)$ is 1 if and only if the design is modified rotatable design and it is smaller than one for a nearly modified rotatable design.

Table1 . Measure of modified rotatability for second order response surface designs using BIBD.

(3,3,2,2,1), $N=50$, $y_1=2$, $y_2=1$, $n_0=20$, $a=1.414214$				
a	c	g	$R_v(D)$	$P_v(D)$
1	2.25	1	0.5865	0.6303
1.3	2.7140	0.7692	0.3699	0.7299
*1.414214	3	0.7071	0	1
1.6	3.6384	0.625	4.0965	0.1962
1.9	5.2580	0.6148	22.4483	0.0426
2.2	7.8564	0.6148	40.0477	0.0244
2.5	11.7656	0.6148	52.9211	0.0185
2.8	17.3664	0.6148	61.5079	0.0159
3.1	25.0880	0.6148	67.1201	0.0147

(4,4,3,3,2), $N=81$, $y_1=1$, $y_2=3$, $n_0=25$, $a=1.414214$				
a	c	g	$R_v(D)$	$P_v(D)$
1	1.875	1	0.16875	0.8556
1.3	2.5710	0.7692	0.0621	0.9415
*1.414214	3	0.7071	0	1
1.6	3.9576	0.6580	0.3044	0.7666
1.9	6.3870	0.6580	1.1479	0.4656
2.2	10.2846	0.6580	1.7875	0.3587
2.5	16.1484	0.6580	2.1876	0.3137
2.8	24.5496	0.6580	2.4314	0.2914
3.1	36.1320	0.6580	2.5825	0.2791

(5,10,6,3,3), N= 150, y_1 =1, y_2= 3, n_0= 40, a = 1.414214				
a	c	g	$R_v(D)$	$P_v(D)$
1	2.25	1	0.0149	0.9854
1.3	2.7140	0.7692	0.0094	0.9907
*1.414214	3	0.7071	0	1
1.6	3.6384	0.625	0.1042	0.9056
1.9	5.2580	0.5578	1.2443	0.4456
2.2	7.8564	0.5578	2.2199	0.3106
2.5	11.7656	0.5578	2.9335	0.2542
2.8	17.3664	0.5578	3.4095	0.2268
3.1	25.0880	0.5578	3.7206	0.2118

(6,10,5,3,2), N= 121, y_1 =1, y_2= 1, n_0= 29, a = 1.414214				
a	c	g	$R_v(D)$	$P_v(D)$
1	2.625	1	0.0044	0.9957
1.3	2.8570	0.7692	0.004	0.9960
*1.414214	3	0.7071	0	1
1.6	3.3192	0.625	0.0667	0.9374
1.9	4.1290	0.5263	1.8138	0.3554
2.2	5.4282	0.4545	13.5355	0.0688
2.5	7.3828	0.4428	26.1461	0.0368
2.8	10.1832	0.4429	33.9292	0.0286
3.1	14.0440	0.4429	39.7519	0.0245

(7,7,4,4,2), N= 162, y_1 =1, y_2= 1, n_0= 36, a = 2				
a	c	g	$R_v(D)$	$P_v(D)$
1	2.0625	1	0.0138	0.9864
1.3	2.1785	0.7692	0.0704	0.9342
1.6	2.4096	0.625	0.1339	0.8819
1.9	2.8145	0.5263	0.0315	0.9694
*2	3	0.5	0	1
2.2	3.4641	0.4545	0.3459	0.7430
2.5	4.4414	0.4	4.7560	0.1737
2.8	5.8416	0.3571	23.1226	0.0415
3.1	7.7720	0.3226	75.2463	0.0131

(8,14,7,4,3), N= 300, $y_1 = 1$, $y_2 = 1$, $n_0 = 60$, a = 2

a	c	g	$R_v(D)$	$P_v(D)$
1	2.375	1	0.0014	0.9986
1.3	2.4523	0.7692	0.0078	0.9922
1.6	2.6064	0.625	0.0175	0.9828
1.9	2.8763	0.5263	0.005	0.9950
*2	3	0.5	0	1
2.2	3.3094	0.4545	0.06703	0.9372
2.5	3.9609	0.4	1.0937	0.4776
2.8	4.8944	0.3571	6.084	0.1411
3.1	6.1813	0.3535	10.5089	0.0869

(9,18,8,4,3), N= 726, $y_1 = 2$, $y_2 = 1$, $n_0 = 132$, a = 2

a	c	g	$R_v(D)$	$P_v(D)$
1	2.6875	1	0.0021	0.9998
1.3	2.7262	0.7692	0.0017	0.9988
1.6	2.8032	0.625	0.0315	0.9695
1.9	2.9382	0.5263	0.001	0.8480
*2	3	0.5	0	1
2.2	3.1547	0.4545	0.0166	0.9837
2.5	3.4805	0.4	0.3357	0.7487
2.8	3.9472	0.3571	2.2882	0.3041
3.1	4.5907	0.3226	9.8146	0.0925

(10,18,9,5,4), N= 441, $y_1 = 1$, $y_2 = 6$, $n_0 = 33$, a = 1.414214

a	c	g	$R_v(D)$	$P_v(D)$
1	2.4375	1	0.0004	0.9996
1.3	2.7855	0.7692	0.0003	0.9997
*1.414214	3	0.7071	0	1
1.6	3.4788	0.625	0.0045	0.9955
1.9	4.6935	0.5263	0.1017	0.9077
2.2	6.6423	0.4671	0.5745	0.6351
2.5	9.5742	0.4617	0.8105	0.5523
2.8	13.7748	0.4617	0.9807	0.5048
3.1	19.5660	0.4617	1.0975	0.4767

(11,11,5,5,2), N= 242, y_1 =1, y_2 = 2, n_0 = 22, a = 1.414214				
A	c	g	$R_v(D)$	$P_v(D)$
1	2.625	1	0.0025	0.9976
1.3	2.8570	0.7692	0.0022	0.9978
1.414214	3	0.7071	0	1
1.6	3.3192	0.625	0.0375	0.9638
1.9	4.1290	0.5263	1.0193	0.4952
2.2	5.4282	0.4545	7.6066	0.1162
2.5	7.3828	0.4111	26.6264	0.0362
2.8	10.1832	0.4111	34.5524	0.02812
3.1	14.044	0.4111	40.482	0.02411

*indicates modified rotatability value using BIBD. (cf. Victorbabu and Vasundharadevi (2005))

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