Original Research Article

Inverted Power Rama distribution with applications to life time data

Abstract

In this paper, we introduced the Inverted Power Rama distribution as an extension of the Inverted Rama distribution. This new distribution is capable of modeling real life data with upside down bathtub shape and heavy tailed data. Mathematical and statistical characteristics such as the quantile function, mode, moments and moment generating function, entropy measure, stochastic ordering and order statistics have been derived. Furthermore, reliability functions like survival function, hazard function, and odds function have been derived. The method of maximum likelihood was used for estimating the parameters of the distribution. To demonstrate the applicability of the distribution, a numerical example was given. Based on the results, the proposed distribution performed better than the competing distributions.

Keywords: Rama distribution, Inverted Rama distribution, Inverted Power Rama distribution, Life time distribution, Order statistics, Goodness of fit

2010 Mathematics Subject Classification: 53C25, 83C05, 57N16

1. Introduction

In many statistical investigations interest lies in conducting life time data analysis. The modeling of lifetime data depends heavily on the behaviour of the hazard rate. Many life time data have monotone (increasing and non-increasing) hazard rates while some have non-monotone (bathtub and upside-down bathtub or unimodal) hazard rates. Several statistical distributions exist for modeling lifetime data. The Rama distribution introduced by [1] is one of the popular statistical distributions used in modeling lifetime data in biomedical science and engineering. The Rama distribution is specifically used for lifetime data with monotone hazard rates. In practice, however, the Rama distribution cannot be used to appropriately model data with non-monotone hazard rates. As at the time of the conduct of this study, many research works have been done with the aim of developing better extensions of the Rama distribution. For instance, [2] developed a two parameter Rama distribution which is more flexible for modeling life time data. Also, [3] developed a weighted two-parametric Rama distribution for modeling lifetime data. Further, the work by [4] proposed a two-parameter power Rama distribution and applied to modeling lifetime data.

Undoubtedly, the Rama distribution and its extensions reviewed in this paper do not provide a reasonable fit for lifetime data with non-monotone hazard rates, such as the upside-down bathtub hazard rates, which are common in many statistical investigations. For example, the lifetime models that present upside-down bathtub hazard rates curves can be observed when modeling a disease whose mortality reaches a peak after some finite period and then declines gradually. The need for extended forms of the Rama distribution to capture lifetime data with non-monotone upside-down bathtub hazard rates arises in many applied areas. Other forms of statistical distributions that have been used by researchers to model lifetime data exhibiting upside-down bathtub hazard rate are those of [5], [6], [7], [8], [9],[10] and [11] among others.

The aim of this article is to introduce an inverted power Rama distribution that fits well, lifetime data with upside-down bathtub hazard rate and however, a skewed data. The rest of the paper is organized as follows. In Section 2, some properties of the inverted power Rama (IPR) distribution are derived. Section 3 deals with reliability analyses such as survival function, hazard rate function and odds function. Section 4 deals with the maximum likelihood estimation of the parameters of the IPR distribution, derivation of the Fisher Information matrix and construction of confidence intervals for the parameters of the distribution. The analysis of real data set is presented in Section 5. Finally, in Section 6, we conclude the paper.

To derive the inverted power Rama distribution, we recall that for a random variable Y , [1] defines the probability density function (PDF) of the Rama distribution as

$$f_{RD}\left(y;\theta\right)=\frac{\theta^{4}}{\theta^{3}+6}\left(1+y^{3}\right)e^{-\theta y};\ y,\theta>0 \tag{1}$$

Using the transformation $X=g\left(X\right)=Y^{-\frac{1}{\alpha}}$ in (1), one obtains the PDF of the inverted Power Rama distributed random variable X as

$$f_{IPR}\left(x;\alpha,\theta\right)=\frac{\alpha\theta^{4}}{\theta^{3}+6}\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{-\alpha}};\ x,\alpha,\theta>0 \tag{2}$$

The corresponding cumulative density function (CDF) of the inverted Rama distributed random variable X is

$$F_{IPR}\left(x;\alpha,\theta\right)=\left[1+\frac{\theta^{3}x^{-3\alpha}+3\theta^{2}x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^{3}+6}\right]e^{-\theta x^{-\alpha}};x,\alpha,\theta>0 \tag{3}$$

It may be noted that when $\alpha=1$, the proposed distribution reduces to the Inverted Rama distribution with PDF given by

$$f_{IR}\left(x;\theta\right)=\frac{\theta^{4}}{\theta^{3}+6}\left(\frac{1+x^{3}}{x^{5}}\right)e^{-\frac{\theta}{x}};\ x,\theta>0 \tag{4}$$

Figures 1a,1b, 1c and 1d show the pdf and cdf plots of the inverted power Rama distribution for varying values of α and θ

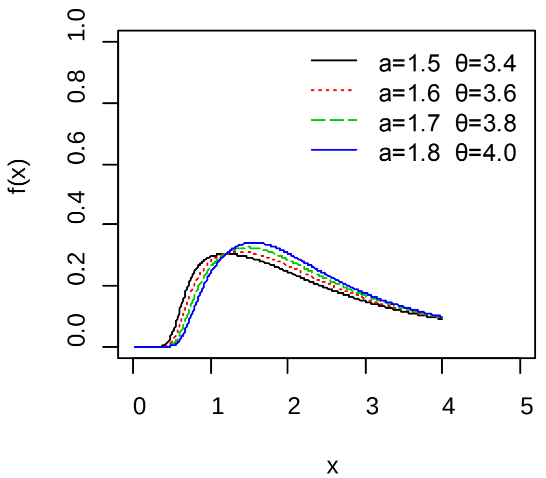

Figure 1a: pdf plot of IPR

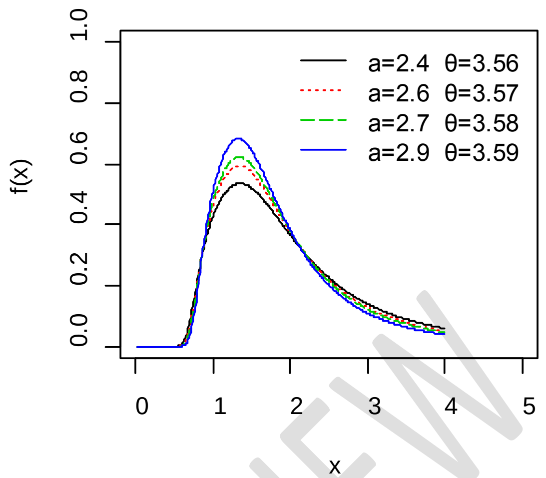

Figure 1b: pdf plot of IPR

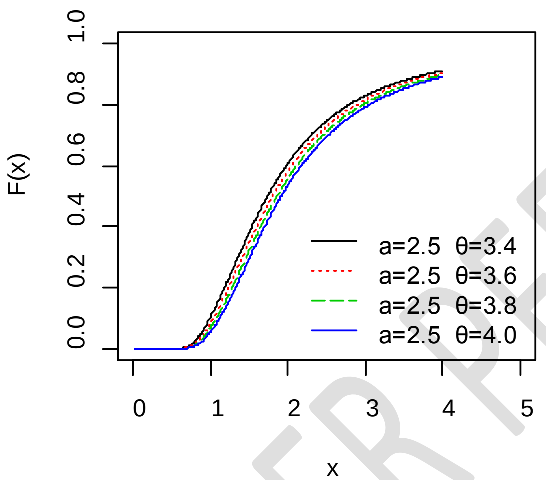

Figure 1c: cdf plot of IPR

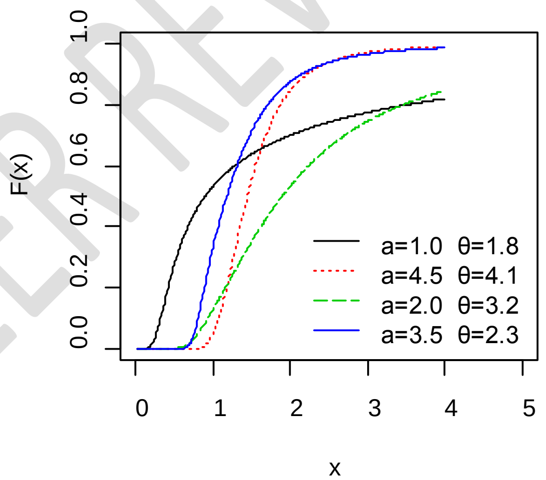

Figure 1d: cdf plot of IPR

2. Mathematical Characteristics

2.1 Mode of the Inverted Power Rama Distribution

The mode is useful in determining the shape of the distribution. So, for the proposed distribution, the first derivative of $f(x)$ is obtained from (2) as follows

$$f'_{IPR}(x) = \begin{bmatrix} x^{-\alpha-1}\left(\theta\alpha x^{-\alpha-1}\right)e^{-\theta x^{-\alpha}} + e^{-\theta x^{-\alpha}}\left(-\alpha-1\right)x^{-\alpha-2} \\ +x^{-4\alpha-1}\left(\theta\alpha x^{-\alpha-1}\right)e^{-\theta x^{-\alpha}} + e^{-\theta x^{-\alpha}}\left(-4\alpha-1\right)x^{-4\alpha-2} \end{bmatrix} \tag{5}$$

Letting $b = x^{-\alpha}$ in (5), we have

$$f'_{IPR}(x;\alpha,\theta) = \frac{\alpha\theta^4}{\theta^3+6}e^{-\theta x^{-\alpha}}x^{-(\alpha+2)}\eta(b) \tag{6}$$

where $\eta(b) = \left(\alpha\theta b^4 - (4\alpha+1)b^2 + \alpha\theta b - (\alpha+1)\right)$ (7)

If we let $\eta(b) = 0$, the positive root of (7), gives the mode of the IPR distribution. To observe the asymptotic behavior, the limit of $f(x)$ is evaluated at $x = 0$ and $x = \infty$ respectively

$$\lim_{x\to 0} f_{IPR}(x;\alpha,\theta) = \lim_{x\to 0}\left\{\frac{\alpha\theta^4}{\theta^3+6}\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{\alpha}}\right\} = 0 \quad (8)$$

$$\lim_{x\to +\infty} f_{IPR}(x;\alpha,\theta) = \lim_{x\to \infty}\left\{\frac{\alpha\theta^4}{\theta^3+6}\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{\alpha}}\right\} = 0 \quad (9)$$

Since $\lim_{x\to 0} f'_{IPR}(x;\alpha,\theta) = 0$ and $\lim_{x\to +\infty} f'_{IPR}(x;\alpha,\theta) = 0$, the inverted power Rama distribution is unimodal. To further support this claim, it is also observed that

$$\lim_{x\to \infty} F_{IPR}(x;\alpha,\theta) = 1$$

2.2 Quantile function

The quantile function is significant for random number generation. It can also be used in finding percentiles of a distribution. Quantile function is defined by;

$$u = F(x) \quad (10)$$

where U is distributed as uniform distribution, $U \sim [0,1]$ and $F(x)$, is the CDF of a distribution.

Proposition 2.2 Let X be a random variable having the PDF of an IPR distribution, then the quantile $Q(p)$ function is

$$x = \left\{\frac{1}{\theta}\ln\left[\frac{1}{Q(p)} + \frac{\theta^3 x^{-3\alpha} + 3\theta^2 x^{-2\alpha} + 6\theta x^{-\alpha}}{Q(p)\left(\theta^3+6\right)}\right]\right\}^{-\frac{1}{\alpha}} \quad (11)$$

Proof. To proof proposition 2.2, recall that a quantile function $Q(p)$ satisfies the equation;

$$Q(p) = F_{IPR}(x;\alpha,\theta) \quad (12)$$

Where $Q(p) \sim [0,1]$ and $F_{IPR}(x;\alpha,\theta)$ is the CDF of Inverted Power Rama distribution. Thus,

$$Q(p) = \left[1 + \frac{\theta^3 x^{-3\alpha} + 3\theta^2 x^{-2\alpha} + 6\theta x^{-\alpha}}{\theta^3+6}\right]e^{-\theta x^{-\alpha}} \quad (13)$$

$$e^{-\theta x^{-\alpha}} = \frac{1}{Q(p)}\left[1 + \frac{\theta^3 x^{-3\alpha} + 3\theta^2 x^{-2\alpha} + 6\theta x^{-\alpha}}{\theta^3+6}\right] \quad (14)$$

$$x=\left\{\frac{1}{\theta}\ln\left[\frac{1}{Q(p)}+\frac{\theta^{3}x^{-3\alpha}+3\theta^{2}x^{-2\alpha}+6\theta x^{-\alpha}}{Q(p)\left(\theta^{3}+6\right)}\right]\right\}^{-\frac{1}{\alpha}} \tag{15}$$

Eq.(15) completes the proof of the quantile of the Inverted Power Rama distribution.

2.3 Moments

Several fascinating characteristics of a distribution can be studied via themoments. For instance, measure of central tendency, dispersion, coefficient of skewness and coefficient of kurtosis. Consequently, it is essential to derive the moments for any new distribution proposed.

Proposition 2.3 Given a random variable X from IPR distribution, the rth crude moment $E\left(X^{r}\right)$ is given by

$$E\left(X^{r}\right)=\theta^{\frac{r}{\alpha}}\Gamma\left(\frac{\alpha-r}{\alpha}\right)\left[\frac{\alpha^{3}\theta^{3}+6\alpha^{3}-11\alpha^{2}r+6\alpha r^{2}-r^{3}}{\alpha^{3}\left(\theta^{3}+6\right)}\right];\alpha>1 \tag{16}$$

Proof. By definition, the r^{th} moment about the origin is

$$E\left(X^{r}\right)=\int_{0}^{\infty}x^{r}f_{IPR}\left(x;\alpha,\theta\right)dx \tag{17}$$

$$=\frac{\alpha\theta^{4}}{\theta^{3}+6}\int_{0}^{\infty}x^{r}\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{-\alpha}}dx$$

$$=\frac{\alpha\theta^{4}}{\theta^{3}+6}\left[\int_{0}^{\infty}x^{-\alpha+r-1}e^{-\theta x^{-\alpha}}dx+\int_{0}^{\infty}x^{-4\alpha+r-1}e^{-\theta x^{-\alpha}}dx\right] \tag{18}$$

Letting $y=x^{-\alpha}$, $\Rightarrow x=y^{\frac{1}{\alpha}}$ in (18) and applying little algebra gives

$$E\left(X^{r}\right)=\frac{\theta^{4}}{\theta^{3}+6}\left[\int_{0}^{\infty}\frac{e^{-\frac{\theta}{y}}}{y^{1-\frac{r}{\alpha}+1}}dy+\int_{0}^{\infty}\frac{e^{-\frac{\theta}{y}}}{y^{4-\frac{r}{\alpha}+1}}dy\right] \tag{19}$$

Using the fact that $\int_{0}^{\infty}\frac{e^{-\beta/x}}{x^{\alpha+1}}dx=\frac{\Gamma(\alpha)}{\beta^{\alpha}}$ one manipulates Eq. (19) to obtain

$$E\left(X^{r}\right)=\frac{\theta^{4}}{\theta^{3}+6}\left[\frac{\Gamma\left(1-\frac{r}{\alpha}\right)}{\theta^{\left(1-\frac{r}{\alpha}\right)}}+\frac{\Gamma\left(4-\frac{r}{\alpha}\right)}{\theta^{\left(4-\frac{r}{\alpha}\right)}}\right] \tag{20}$$

$$E\left(X^{r}\right)=\theta^{\frac{r}{\alpha}}\Gamma\left(\frac{\alpha-r}{\alpha}\right)\left[\frac{\alpha^{3}\theta^{3}+6\alpha^{3}-11\alpha^{2}r+6\alpha r^{2}-r^{3}}{\alpha^{3}\left(\theta^{3}+6\right)}\right] \tag{21}$$

Eq. (21) completes the proof the rth crude moment of the IPR distribution.

The mean of the IPR distribution is obtained by setting $r=1$ in (21). Thus,

$$E(X)=\theta^{\frac{1}{\alpha}}\Gamma\left(\frac{\alpha-1}{\alpha}\right)\left[\frac{\alpha^3\theta^3+6\alpha^3-11\alpha^2+6\alpha-1}{\alpha^3\left(\theta^3+6\right)}\right] \tag{22}$$

For $r=2$ in (21), the second crude moment $E\left(X^2\right)$ of the IPR distribution becomes

$$E\left(X^2\right)=\theta^{\frac{2}{\alpha}}\Gamma\left(\frac{\alpha-2}{\alpha}\right)\left[\frac{\alpha^3\theta^3+6\alpha^3-22\alpha^2+24\alpha-8}{\alpha^3\left(\theta^3+6\right)}\right] \tag{23}$$

The variance of the IPR distribution is obtained as follows

$$Var(X)=E\left(X^2\right)-\left(E(X)\right)^2$$

$$=\theta^{\frac{2}{\alpha}}\Gamma\left(\frac{\alpha-2}{\alpha}\right)\left(\frac{\alpha^3\theta^3+6\alpha^3-22\alpha^2+24\alpha-8}{\alpha^3\left(\theta^3+6\right)}\right)-\left[\theta^{\frac{1}{\alpha}}\Gamma\left(\frac{\alpha-1}{\alpha}\right)\left(\frac{\alpha^3\theta^3+6\alpha^3-11\alpha^2+6\alpha-1}{\alpha^3\left(\theta^3+6\right)}\right)\Gamma\left(\frac{\alpha-1}{\alpha}\right)\right]^2 \tag{24}$$

2.4 Moment generating function of the inverted power Rama distribution

Proposition 2.4 Give a random variable X that follows Inverted Power Rama distribution, the moment generating function is given by;

$$M_x(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\theta^{\frac{r}{\alpha}}\Gamma\left(\frac{\alpha-r}{\alpha}\right)\left[\frac{\alpha^3\theta^3+6\alpha^3-11\alpha^2r+6\alpha r^2-r^3}{\alpha^3\left(\theta^3+6\right)}\right]$$

Proof. Let X be a random variable from a continuous univariate distribution, the moment generating function is defined as

$$M_X(t)=E\left(e^{tX}\right)=\int_0^{\infty}e^{tx}f(x)\,dx \tag{25}$$

$$=\int_0^{\infty}\left(1+tx+\frac{(tx)^2}{2!}+\cdots\right)f(x)\,dx \tag{26}$$

$$=\sum_{r=0}^{\infty}\frac{t^r}{r!}E\left(X^r\right) \tag{27}$$

Substituting for $E\left(X^r\right)$, we obtain an expression for the moment generating function as

$$M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\theta^{\frac{r}{\alpha}}\Gamma\left(\frac{\alpha-r}{\alpha}\right)\left[\frac{\alpha^3\theta^3+6\alpha^3-11\alpha^2r+6\alpha r^2-r^3}{\alpha^3\left(\theta^3+6\right)}\right] \tag{28}$$

2.5 Entropy Measure

Entropy is very useful in determining the uncertainty of a distribution**.** Entropy has applications in economics, probability and statistics, communication theory etc. Large value of entropy signifies large uncertainty in the data. In this section, we derive an expression for the Rényi entropy of the IPR distribution.

Proposition 2.5 Suppose X is a random variable having the PDF of IPR distribution, the Rényi entropy is given by

$$J_R(\gamma)=\frac{1}{1-\gamma}log\left[(\alpha-1)\sum_{q=0}^{\infty}\binom{\gamma}{q}\left(\frac{\alpha\theta^4}{\theta^3+6}\right)^{\gamma}\Gamma\left(4\gamma+\frac{\gamma}{\alpha}-\frac{1}{\alpha}-3q\right)(\theta\gamma)^{-\left(4\gamma+\frac{\gamma}{\alpha}-\frac{1}{\alpha}-3q\right)}\right]$$

Proof. The Rényi entropy of a random variable X from a continuous distribution is given by

$$J_R(\gamma)=\frac{1}{1-\gamma}\log\left[\int_R f^{\gamma}(x;\alpha,\theta)\,dx\right];\gamma>0,\gamma\neq 1 \tag{29}$$

$$=\frac{1}{1-\gamma}\log\left\{\int_0^{\infty}\left[\frac{\alpha\theta^4}{\theta^3+6}\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{-\alpha}}\right]^{\gamma}dx\right\} \tag{30}$$

$$=\frac{1}{1-\gamma}\log\left\{\left(\frac{\alpha\theta^4}{\theta^3+6}\right)^{\gamma}\int_0^{\infty}\left(1+x^{3\alpha}\right)^{\gamma}x^{-\gamma(4\alpha+1)}e^{-\theta\gamma x^{-\alpha}}dx\right\} \tag{31}$$

To simplify (31), we utilize the binomial expansion $\left(1+x^{3\alpha}\right)^{\gamma}=\sum_{q=0}^{\infty}\binom{\gamma}{q}x^{3\alpha q}$. Consequently, we have

$$J_R(\gamma)=\frac{1}{1-\gamma}\log\left\{\sum_{q=0}^{\infty}\binom{\gamma}{q}\left(\frac{\alpha\theta^4}{\theta^3+6}\right)^{\gamma}\int_0^{\infty}x^{3\alpha q-4\alpha\gamma-\gamma}e^{-\frac{\theta\gamma}{x^{\alpha}}}dx\right\} \tag{32}$$

Letting $y=x^{\alpha}$ in (32) and simplifying yields

$$J_R(\gamma)=\frac{1}{1-\gamma}\log\left\{\alpha^{-1}\sum_{q=0}^{\infty}\binom{\gamma}{q}\left(\frac{\alpha\theta^4}{\theta^3+6}\right)^{\gamma}\int_0^{\infty}y^{-\left(4\gamma+\frac{\gamma}{\alpha}-\frac{1}{\alpha}-3q+1\right)}e^{-\frac{\theta\gamma}{y}}dy\right\} \tag{33}$$

Since $\int_0^{\infty}x^{-(\alpha+1)}e^{-\frac{\beta}{x}}dx=\frac{\beta^{\alpha}}{\Gamma(\alpha)}$, Eq. (33) reduces to

$$J_R(\gamma)=\frac{1}{1-\gamma}log\left[(\alpha-1)\sum_{q=0}^{\infty}\binom{\gamma}{q}\left(\frac{\alpha\theta^4}{\theta^3+6}\right)^{\gamma}\Gamma\left(4\gamma+\frac{\gamma}{\alpha}-\frac{1}{\alpha}-3q\right)(\theta\gamma)^{-\left(4\gamma+\frac{\gamma}{\alpha}-\frac{1}{\alpha}-3q\right)}\right] \tag{34}$$

Eq. (34) completes the proof Rényi entropy

2.6 Order Statistics

Suppose $x_1, x_2 ..., x_n$ are random samples of size n from a continuous distribution with PDF and CDF, $f\left(x\right)$ and $F\left(x\right)$ respectively. If these random variables are arranged in ascending order, they are referred to as order statistics. That is, the order statistics is such that $x_1 \le x_2 \le x_3 \le ... \le x_n$. The PDF of the ω^{th} order statistics is

$$f_X\left(x\right) = \frac{n!}{\left(\omega-1\right)!\left(n-\omega\right)!} F^{\omega-1}\left(x\right)\left(1-F\left(x\right)\right)^{n-\omega} f\left(x\right) \tag{35}$$

$$= \sum_{j=0}^{n-\omega} \frac{n!}{\left(\omega-1\right)!\left(n-\omega\right)!} \binom{n-\omega}{j} \left(-1\right)^j F^{\omega-1}\left(x\right) F^j\left(x\right) f\left(x\right)$$

$$= \frac{n!}{\left(\omega-1\right)!\left(n-\omega\right)!}\left(x\right) \sum_{j=0}^{n-\omega} \binom{n-\omega}{j} \left(-1\right)^j F^{\omega+j-1}\left(x\right) f\left(x\right) \tag{36}$$

Substituting (2) and (3) in (36), we have

$$f_X\left(x\right) = \frac{n!}{\left(\omega-1\right)!\left(n-\omega\right)!} \sum_{j=0}^{n-\omega} \binom{n-\omega}{j} \left(-1\right)^j \left\{\left[1+\frac{\theta^3 x^{-3\alpha}+3\theta^2 x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right] e^{-\theta x^{-\alpha}}\right\}^{\omega+j-1} \times \left\{\frac{\alpha\theta^4}{\theta^3+6}\left(1+x^{-3\alpha}\right) x^{-\alpha-1} e^{-\theta x^{-\alpha}}\right\} \tag{37}$$

$$= \left\{\begin{array}{l} \dfrac{\alpha\theta^4 n!\left(1+x^{-3\alpha}\right) x^{-\alpha-1} e^{-\theta x^{-\alpha}}}{\left(\theta^3+6\right)\left(\omega-1\right)!\left(n-\omega\right)!} \displaystyle\sum_{j=0}^{n-\omega} \binom{n-\omega}{j} \left(-1\right)^j \\ \times \left\{\left[1+\dfrac{\theta^3 x^{-3\alpha}+3\theta^2 x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right] e^{-\theta x^{-\alpha}}\right\}^{\omega+j-1} \end{array}\right\} \tag{38}$$

Using binomial series, the expression

$$\left\{\left[1+\frac{\theta^3 x^{-3\alpha}+3\theta^2 x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right] e^{-\theta x^{-\alpha}}\right\}^{\omega+j-1} = \sum_{k=0}^{\infty}\sum_{l=0}^{k}\sum_{m=0}^{l} \binom{\omega+j-1}{k}\binom{k}{l}\binom{l}{m} \frac{3^l 2^m \theta^{3k-l-m}}{\left(\theta^3+6\right)^k} x^{-\alpha\left(3k-l-m\right)} e^{-k\theta x^{-\alpha}} \tag{39}$$

Substituting (39) into (38), we have the ωth PDF of the order statistics of the IPR distribution as

$$f_X\left(x\right) = \frac{\alpha n!\left(1+x^{-3\alpha}\right) e^{-\theta\left(1+k\right)x^{-\alpha}}}{\left(\omega-1\right)!\left(n-\omega\right)!\left(\theta^3+6\right)^{k+1}} \sum_{j=0}^{n-\omega} \binom{n-\omega}{j} \left(-1\right)^j \sum_{k=0}^{\infty} \binom{\omega+j-1}{k} \sum_{l=0}^{k} \binom{k}{l} \sum_{m=0}^{l} \binom{l}{m} \times 3^l 2^m \theta^{3k-l-m+4} x^{-\alpha\left(3k-l-m+1\right)-1} \tag{40}$$

The corresponding CDF, $F_X\left(x\right)$ of the order statistics of the IPR distribution is obtained as follows;

$$F_X\left(x\right)=\sum_{u=\omega}^{n}\binom{n}{u}F^{u}\left(x\right)\left(1-F\left(x\right)\right)^{n-u} \tag{41}$$

$$=\sum_{u=\omega}^{n}\sum_{v=0}^{n-u}\binom{n}{u}\binom{n-u}{v}\left(-1\right)^{v}F^{u+v}\left(x\right) \tag{42}$$

On substituting for Eq. 3, we have;

$$F_X\left(x\right)=\sum_{u=\omega}^{n}\sum_{v=0}^{n-u}\binom{n}{u}\binom{n-u}{v}\left(-1\right)^{v}\left\{\left[1+\frac{\theta^3x^{-3\alpha}+3\theta^2x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right]e^{-\theta x^{-\alpha}}\right\}^{u+v} \tag{43}$$

Using binomial expansion,

$$\left\{\left[1+\frac{\theta^3x^{-3\alpha}+3\theta^2x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right]e^{-\theta x^{-\alpha}}\right\}^{u+v}$$

$$=\sum_{a=0}^{\infty}\sum_{s=0}^{a}\sum_{z=0}^{s}\binom{u+v}{a}\binom{a}{s}\binom{s}{z}\frac{3^s2^z\theta^{3a-s-z}}{\left(\theta^3+6\right)^a}x^{-\alpha(3a-s-z)-a\theta x^{-\alpha}} \tag{44}$$

Substituting (44) in (43) and simplifying, we have

$$F_X\left(x\right)=\sum_{u=\omega}^{n}\sum_{v=0}^{n-u}\sum_{a=0}^{\infty}\sum_{s=0}^{a}\sum_{z=0}^{s}\binom{n}{u}\binom{n-u}{v}\binom{u+v}{a}\binom{a}{s}\left(-1\right)^{v}\binom{s}{z}\frac{3^s2^z\theta^{3a-s-z}}{\left(\theta^3+6\right)^a}x^{-\alpha\left(3a-s-z\right)}e^{-a\theta x^{-\alpha}} \tag{45}$$

2.7 Stochastic ordering

Stochastic ordering is an essential tool for quantifying the behavior of random variables in terms of their sizes. Given that X and Y are distributed according to Eq. (2). Let $f_X\left(x;\alpha,\theta\right)$, $f_Y\left(x;\alpha,\theta\right)$ and $F_X\left(x;\alpha,\theta\right)$, $F_Y\left(x;\alpha,\theta\right)$ denote the probability density function and distribution function of X and Y respectively. The random variable X is said to be smaller than a random variable Y in the

a) Stochastic order $\left(X\leq_{st}Y\right)$ if $F_X\left(x\right)\geq F_Y\left(x\right);\forall x$
b) Hazard rate order $\left(X\leq_{hr}Y\right)$ if $h_X\left(x\right)\geq h_Y\left(x\right);\forall x$
c) Mean residual life order $\left(X\leq_{mrl}Y\right)$ if $m_X\left(x\right)\geq m_Y\left(x\right);\forall x$
d) Likelihood ratio order $\left(X\leq_{lr}Y\right)$ if $\dfrac{f_X\left(x;\alpha,\theta\right)}{f_Y\left(x;\alpha,\theta\right)}$ decreases in x

These results were established by [12]. The order of the distributions is as follows

$$X\leq_{lr}Y\Rightarrow X\leq_{hr}Y\Rightarrow X\leq_{mrl}Y\Rightarrow X\leq_{st}Y$$

The IPR distribution is ordered based on the distribution with the strongest likelihood ratio, as showing in proposition 4 below

Proposition 2.6 Suppose $X \sim IPR(\alpha_1, \theta_1)$ and $Y \sim IPR(\alpha_2, \theta_2)$. If $\alpha_1 = \alpha_2$ and $\theta_2 \geq \theta_1$, then $X \leq_{lr} Y$. Hence, $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$

Proof. The likelihood ratio is

$$\frac{f_X(x;\alpha_1,\theta_1)}{f_Y(x;\alpha_2,\theta_2)} = \frac{\frac{\alpha_1\theta_1^4}{\theta_1^3+6}\left(1+x^{-3\alpha_1}\right)x^{-\alpha_1-1}e^{-\theta_1 x^{-\alpha_1}}}{\frac{\alpha_2\theta_2^4}{\theta_2^3+6}\left(1+x^{-3\alpha_2}\right)x^{-\alpha_2-1}e^{-\theta_2 x^{-\alpha_2}}} \tag{46}$$

$$= \frac{\left(\alpha_1\theta_1^4\right)\left(\theta_2^3+6\right)\left(1+x^{3\alpha_1}\right)}{\left(\alpha_2\theta_2^4\right)\left(\theta_1^3+6\right)\left(1+x^{3\alpha_2}\right)} x^{4(\alpha_2-\alpha_1)} e^{\theta_2 x^{-\alpha_2} - \theta_1 x^{-\alpha_1}} \tag{47}$$

If $\alpha_1 = \alpha_2 = \alpha$, $\frac{f_X(x;\alpha_1,\theta_1)}{f_Y(x;\alpha_2,\theta_2)} = \left(\frac{\theta_1^4}{\theta_2^4}\right)\left(\frac{\theta_2^3+6}{\theta_1^3+6}\right)e^{(\theta_2-\theta_1)x^{-\alpha}}$

Observe that this is a decreasing function in x when $\theta_2 \geq \theta_1$. Hence, $X \leq_{lr} Y$ and $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

3 Reliability Analyses

3.1 Survival function

Survival function $S(x)$ is the probability that the survival time is greater than or equal to x. In engineering, it is the probability that an item does not fail prior to some time, x. We use survival function in reliability analysis to determine the survival time of items. Let X be a continuous random Variable with CDF, $F(x)$, the survival function of X is

$$S(x) = 1 - F(x) \tag{48}$$

Thus, the survival function of inverted power Rama distribution is

$$S(x) = 1 - \left[1 + \frac{\theta^3 x^{-3\alpha} + 3\theta^2 x^{-2\alpha} + 6\theta x^{-\alpha}}{\theta^3+6}\right] e^{-\theta x^{-\alpha}} \tag{49}$$

3.2 Hazard function

Hazard function, also known as failure rate is the probability that an individual dies at time x given that the individual have survived to that time x. Hazard function is extensively used to express the risk of an event (example, death) occurring at some time t.

Given a random variable X from a continuous distribution, the hazard rate $h(x)$ is given by

$$h(x) = \frac{f(x)}{1-F(x)} \tag{50}$$

Inserting the value of $f(x)$ and $F(x)$ into Eq. (50), we obtain the hazard rate of inverted power Rama distribution as

$$h(x)=\frac{\frac{\alpha\theta^{4}}{\theta^{3}+6}\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{-\alpha}}}{1-\left[1+\frac{\theta^{3}x^{-3\alpha}+3\theta^{2}x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^{3}+6}\right]e^{-\theta x^{-\alpha}}} \quad (51)$$

Figure 2a, 2b, 2c and 2d illustrate the survival and hazard functions of the inverted power Rama distribution for different values of α and θ.

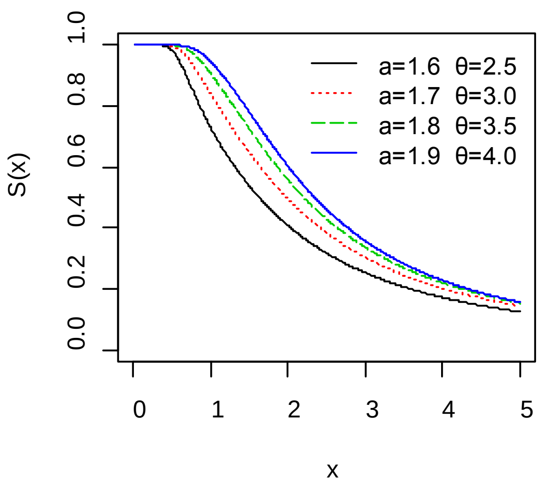

Figure 2a: Survival plot of

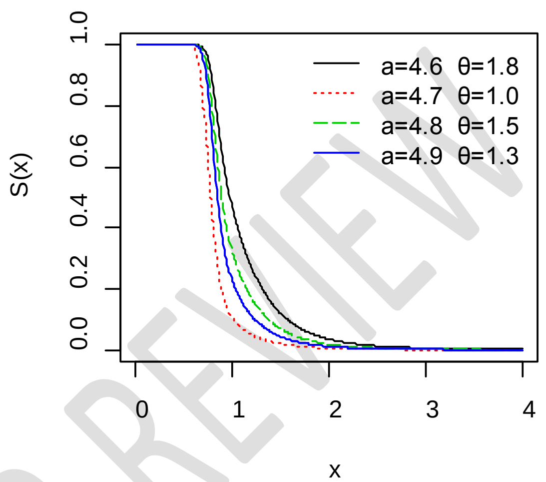

Figure 2b: Survival plot of

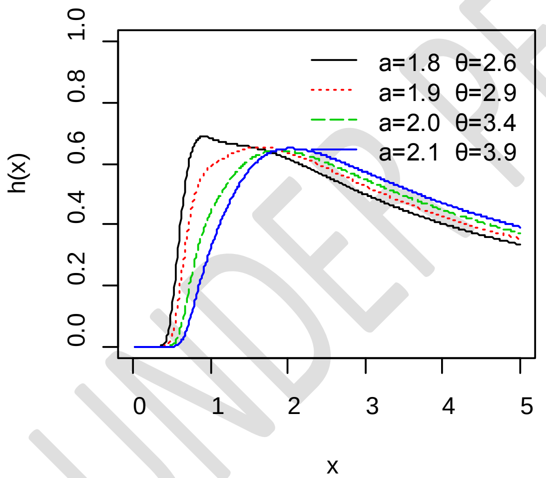

Figure 2c: hazard functior

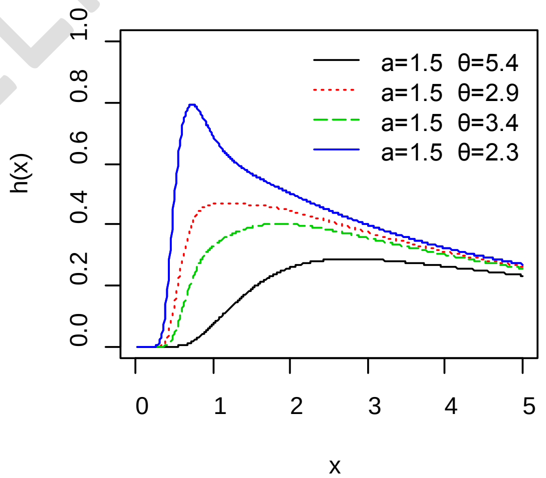

Figure 2d: hazard functior

3.3 Odds Function

Odd rate is an important tool in reliability analysis for modeling real life data set that shows non-monotone hazard rate. Let X be a random variable from a continuous distribution with distribution function $F(x)$ and reliability function $1-F(x)$, the odd function is

$$O(x)=\frac{F(x)}{1-F(x)} \quad (52)$$

Thus, the odds function of the inverted power Rama distribution is given by

$$O(x)=\frac{\left[1+\frac{\theta^3x^{-3\alpha}+3\theta^2x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right]e^{-\theta x^{-\alpha}}}{1-\left[1+\frac{\theta^3x^{-3\alpha}+3\theta^2x^{-2\alpha}+6\theta x^{-\alpha}}{\theta^3+6}\right]e^{-\theta x^{-\alpha}}} \tag{53}$$

4. Maximum Likelihood Estimation

Let $x_1, x_2 ..., x_n$ be a random sample of size n from an inverted power Rama distribution. Then, the log-likelihood (LL) function is

$$LL(\alpha,\theta)=\ln\prod_{i=i}^{n} f\left(x_i;\alpha,\theta\right) \tag{54}$$

$$=\ln\prod_{i=1}^{n}\frac{\alpha\theta^4}{\theta^3+6}\left(1+x^{-3\alpha}\right)x^{-(\alpha+1)}e^{-\theta x^{-\alpha}} \tag{55}$$

$$=\ln\left\{\left(\frac{\alpha\theta^4}{\theta^3+6}\right)^n\prod_{i=1}^{n}\left(1+x_i^{-3\alpha}\right)\prod_{i=1}^{n}x_i^{-(\alpha+1)}e^{-\theta\sum_{i=1}^{n}x_i^{-\alpha}}\right\} \tag{56}$$

$$\begin{aligned}&=n\ln(\alpha)+4n\ln(\theta)-n\ln\left(\theta^3+6\right)\\&+\sum_{i=1}^{n}\ln\left(1+x_i^{-3\alpha}\right)-(\alpha+1)\sum_{i=1}^{n}\ln x_i-\theta\sum_{i=1}^{n}\ln x_i^{-\alpha}\end{aligned} \tag{57}$$

The partial derivatives in terms of the parameters (α,θ) are given as follows

$$\frac{\partial LL}{\partial\alpha}=\frac{n}{\alpha}-3\sum_{i=1}^{n}\frac{x_i^{-3\alpha}lnx_i}{1+x_i^{-3\alpha}}-\sum_{i=0}^{n}lnx_i+\theta\sum_{i=1}^{n}x_i^{-\alpha}lnx_i=0 \tag{58}$$

$$\frac{\partial LL}{\partial\theta}=\frac{4n}{\theta}-\frac{3n\theta^2}{\theta^3+6}-\sum_{i=1}^{n}x_i^{-\alpha}=0 \tag{59}$$

The simultaneous solution of the nonlinear Eqs. (58) and (59) at $\frac{\partial LL}{\partial\alpha}=0$ and $\frac{\partial LL}{\partial\theta}=0$ yields the maximum likelihood estimation of the parameters (α,θ).

In order to estimate the confidence intervals for the parameters using maximum likelihood estimators $\left(\hat{\alpha},\hat{\theta}\right)$, the Fisher information matrix will be used, which for a vector of parameters ξ and $n=1$ are given by the expression

$$\mathrm{I}(\xi)=\mathrm{I}_{i,j}(\xi)=E\left[-\frac{\partial^2}{\partial\xi_i\partial\xi_j}\ln f\left(X\mid\xi\right)\right] \tag{60}$$

Due to the complexity involved in evaluating the information matrix given in (60), the inverse Hessian matrix is used in the maximum likelihood estimates. Consequently, the second-order derivatives of the log-likelihood function are given as follows:

$$\frac{\partial^2 LL}{\partial \alpha^2} = -\frac{n}{\alpha^2} + 9\sum_{i=1}^{n} \frac{x_i^{-3\alpha}\left(\ln x_i\right)^2}{\left(1+x_i^{-3\alpha}\right)^2} - \theta \sum_{i=1}^{n} x_i^{-\alpha}\left(\ln x_i\right)^2 \tag{61}$$

$$\frac{\partial^2 LL}{\partial \theta^2} = -\frac{4n}{\theta^2} + \frac{3n\theta\left(\theta^3-12\right)}{\left(\theta^3+6\right)^3} \tag{62}$$

$$\frac{\partial^2 LL}{\partial \alpha \partial \theta} = \sum_{i=1}^{n} x_i^{-\alpha} \ln x_i \tag{63}$$

$$\frac{\partial^2 LL}{\partial \theta \partial \alpha} = \sum_{i=1}^{n} x_i^{-\alpha} \ln x_i \tag{64}$$

In order to determine the Fisher information matrix for the IPR distribution, the expectations of (61), (62), (63) and (64) are taken, assuming $n=1$. Thus,

$$-E\left[\frac{\partial^2 LL}{\partial \alpha^2}\right] = \frac{n}{\alpha^2} - 9\sum_{i=1}^{n} E\left[\frac{X_i^{-3\alpha}\left(\ln X\right)^2}{\left(1+X_i^{-3\alpha}\right)^2}\right] + \theta \sum_{i=1}^{n} E\left[X_i^{-\alpha}\left(\ln X\right)^2\right] \tag{65}$$

$$E\left[\frac{\partial^2 LL}{\partial \theta^2}\right] = -\frac{4n}{\theta^2} + \frac{3n\theta\left(\theta^3-12\right)}{\left(\theta^3+6\right)^3} \tag{66}$$

$$E\left[\frac{\partial^2 LL}{\partial \alpha \partial \theta}\right] = \sum_{i=1}^{n} E\left[X_i^{-\alpha} \ln x_i\right] \tag{67}$$

where

$$E\left[X_i^{-\alpha} \ln x_i\right] = \int_0^\infty x^{-\alpha} \ln x f\left(x\right) dx \tag{68}$$

$$= \frac{\alpha\theta^4}{\theta^3+6} \int_0^\infty x^{-\alpha} \ln x \left(1+x^{-3\alpha}\right) x^{-\alpha-1} e^{-\theta x^{-\alpha}} dx \tag{69}$$

Before we proceed, let us introduce some important integral identities as stated in [13]:

$$\int_0^\infty t^{\upsilon-1} \ln t e^{-\lambda t} dt = \frac{\Gamma\left(\upsilon\right)}{\lambda^\upsilon}\left[\psi\left(\upsilon\right) - \ln \lambda\right],\ \upsilon, \lambda > 0 \tag{70}$$

$$\int_0^\infty t^{\upsilon-1} \left(\ln t\right)^2 e^{-\lambda t} dt = \frac{\Gamma\left(\upsilon\right)}{\lambda^\upsilon}\left\{\left[\psi\left(\upsilon\right) - \ln \lambda\right]^2 + \zeta\left(2,\upsilon\right)\right\},\ \upsilon, \lambda > 0 \tag{71}$$

where $\psi(t)=\frac{d}{dt}\ln\Gamma(t)$, is digamma function, $\Gamma(t)$ is a gamma function and $\zeta(z,\upsilon)$ is Riemann's zeta function given by $\zeta(z,\upsilon)=\sum_{m=0}^{\infty}\frac{1}{(\upsilon+m)^2}, z>1, \upsilon\neq 0,-1,-2,...$

Using transformation techniques, we let $y=x^{-\alpha}$ in (69). After simplifying, we have

$$E\left[X_i^{-\alpha}\ln X_i\right]=\frac{\theta^4}{\alpha(\theta^3+6)}\int_0^\infty y^{2-1}(\ln y)e^{-\theta y}dy+\frac{\theta^4}{\alpha(\theta^3+6)}\int_0^\infty y^{5-1}(\ln y)e^{-\theta y}dy \tag{72}$$

Applying (70) to (72) and simplifying, we have

$$E\left[X^{-\alpha}\ln X\right]=\frac{\theta^3\left[\psi(2)-\ln\theta\right]+24\left[\psi(5)-\ln\theta\right]}{\alpha\theta(\theta^3+6)} \tag{73}$$

$$E\left[X^{-\alpha}(\ln X)^2\right]=\int_0^\infty x^{-\alpha}(\ln x)^2 f(x)dx \tag{74}$$

$$=\frac{\alpha\theta^4}{\theta^3+6}\int_0^\infty x^{-\alpha}(\ln x)^2\left(1+x^{-3\alpha}\right)x^{-\alpha-1}e^{-\theta x^{-\alpha}}dx \tag{75}$$

Again, we obtain the expression below

$$E\left[X^{-\alpha}(\ln X)^2\right]=\frac{\theta^4}{\alpha^2(\theta^3+6)}\int_0^\infty y^{2-1}(\ln y)^2e^{-\theta y}dy+\frac{\theta^4}{\alpha^2(\theta^3+6)}\int_0^\infty y^{5-1}(\ln y)^2e^{-\theta y}dy \tag{76}$$

Applying (71) to (76), we obtain expression of the form

$$E\left[X^{-\alpha}(\ln X)^2\right]=\frac{\theta^3\left\{\left[\psi(2)-\ln\theta\right]^2+\zeta(2,2)\right\}+24\left\{\left[\psi(5)-\ln\theta\right]^2+\zeta(2,5)\right\}}{\alpha^2\theta(\theta^3+6)} \tag{77}$$

In line with the same procedures used above, one derives the expression for

$$E\left[\frac{X^{-3\alpha}(\ln X)^2}{\left(1+X^{-3\alpha}\right)^2}\right]=\frac{\theta^4}{\alpha^2(\theta^3+6)}\left\{\left[\psi(1)-\ln\theta\right]^2+\zeta(2,1)-\theta J(\theta)\right\} \tag{78}$$

where $J(\theta)=\int_0^\infty\frac{(\ln t)}{\left(1+t^3\right)}e^{-\theta t}dt$

Now, a substitution of (73), (77) and (78) into (65) and (67), we obtain the following

$$I_{11}=E\left[\frac{\partial^2 LL}{\partial\alpha^2}\right]=-\frac{n}{\alpha^2}+\frac{9n\theta^4}{\alpha^2\left(\theta^3+6\right)}\left\{\left[\psi(1)-\ln\theta\right]^2+\zeta(2,1)-\theta J(\theta)\right\}$$
$$-\frac{n\theta^4\left\{\left[\psi(2)-\ln\theta\right]^2+\zeta(2,2)\right\}+24\left\{\left[\psi(5)-\ln\theta\right]^2+\zeta(2,5)\right\}}{\alpha^2\theta\left(\theta^3+6\right)} \quad (79)$$

$$I_{22}=E\left[\frac{\partial^2 LL}{\partial\theta^2}\right]=-\frac{4n}{\theta^2}+\frac{3n\theta\left(\theta^3-12\right)}{\left(\theta^3+6\right)^3} \quad (80)$$

$$I_{12}=I_{21}=E\left[\frac{\partial^2 LL}{\partial\alpha\partial\theta}\right]=\frac{n\theta^3\left[\psi(2)-\ln\theta\right]+24\left[\psi(5)-\ln\theta\right]}{\alpha\theta\left(\theta^3+6\right)} \quad (81)$$

In order to resolve (79), (80) and (81), we find locally the values of the digamma functions for $\psi(1)$, $\psi(2)$, $\psi(5)$ and Riemann's zeta functions $\zeta(2,1)$, $\zeta(2,2)$, and $\zeta(2,5)$.

The asymptotic distribution of the maximum likelihood estimator $\hat{\xi}$ for ξ under consistency state is given by:

$$\sqrt{n}\left(\hat{\xi}-\xi\right)\rightarrow N\left(0,I^{-1}(\xi)\right)$$

where $I^{-1}(\xi)$ is the inverse Fisher information matrix, defined as;

$$\frac{1}{n}I^{-1}(\xi)=\frac{1}{n}\begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22}\end{pmatrix}^{-1}=\begin{pmatrix} Var(\hat{\alpha}) & Cov(\hat{\alpha},\hat{\theta}) \\ Cov(\hat{\alpha},\hat{\theta}) & Var(\hat{\theta})\end{pmatrix} \quad (82)$$

Having obtained the expression in (82), we can now define the asymptotic $100(1-\tau)\%$ confidence intervals for α and θ as given below;

$$\hat{\alpha}\pm Z_{\frac{\tau}{2}}\sqrt{Var(\hat{\alpha})} \text{ and } \hat{\theta}\pm Z_{\frac{\tau}{2}}\sqrt{Var(\hat{\theta})} \quad (83)$$

where $Var(\hat{\alpha})$ and $Var(\hat{\theta})$ denote the elements of the main diagonal of the variance covariance matrix defined in (82).

5 Numerical Applications

In this section, we present two real life data sets to exhibit the practicality of the proposed model. The first data set is the monthly actual taxes revenue in Egypt from January 2006 to November 2010 used in [14], [15] and [16]. The data (in 1000 million Egyptian pounds) is provided below;

5.9, 20.4, 14.9, 16.2, 17.2, 7.8, 6.1, 9.2, 10.2, 9.6, 13.3, 8.5, 21.6, 18.5, 5.1, 6.7, 17.0, 8.6, 9.7, 39.2, 35.7, 15.7, 9.7, 10, 4.1, 36.0, 8.5, 8.0, 9.2, 26.2, 21.9, 16.7, 21.3, 35.4, 14.3, 8.5, 10.6, 19.1, 20.5, 7.1, 7.7, 18.1,16.5, 11.9, 7, 8.6, 12.5, 10.3, 11.2, 6.1, 8.4, 11, 11.6, 11.9, 5.2, 6.8, 8.9, 7.1, 10.8.

The second dataset represents the relief times of twenty patients receiving an analgesic. It was used by [17], reported by [18]

1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7,
4.1, 1.8, 1.5, 1.2, 1.4, 3.0, 1.7, 2.3, 1.6, 2.0

Tables 1 and 2 below show the estimates of the Inverted power Rama (IPR) distribution and the competing distributions, namely; Inverse Rama (IR) distribution, Rama distribution (RD) and a two parameter power Rama (TPPR) distribution, respectively obtained using the first and second datasets. Comparisons of the estimates, computed using maximum likelihood estimation method was made. To select the best distribution, three criteria were used. The criteria include:

- Akaike information criteria (AIC) is given by

$$AIC = -2\ln L + 2k \quad (84)$$

- Bayesian information criteria (BIC) given by

$$BIC = \ln(n)k - 2\ln(\hat{L}) \quad (85)$$

- Corrected Akaike information criteria(AIC_c) given by

$$AICc = AIC + \frac{2k(k+1)}{n-k-1} \quad (86)$$

where $\hat{L}$ denotes the log-likelihood at Maximum Likelihood Estimates (MLEs), k is the number of parameters in the distribution, and n is the sample size.The distribution with least AIC, BIC, AIC_c and log-likelihood is considered as best. Table 1 and 2 show that IPR distribution has the least values of AIC, BIC, and AIC_c as compared to the competing distributions considered with it. Thus, IPR distribution is considered to provide best fit than IR, RD and TPPR distributions.

Table 1 MLEs, S.E, LL, AIC, BIC and AICc (Data 1)

Model	MLE	S.E	LL	AIC	BIC	AIC_c
IPR	α = 2.2466 θ = 144.6290	0.2225 67.4959	188.9396	381.8792	386.0342	382.0934
RD	θ = 0.2956	0.0192	193.3599	388.7198	390.7974	388.79
TPPR	α = 0.9463 θ = 0.3419	0.0792 0.0763	193.1374	390.2748	394.4298	390.489
IR	θ = 10.5177	1.3291	211.5403	425.0805	427.1581	425.0805

Table 2: MLEs, LL, S.E, AIC, BIC, AICc and C.I of IPR distribution (Data 2)

Model	MLE	S.E	LL	AIC	BIC	AIC_c
IPR	α = 4.1181 θ = 6.6111	0.6671 1.836	15.4089	34.8178	36.8093	35.5237
TPPR	α = 1.7828 θ = 1.0341	0.1972 0.1624	21.4538	46.9076	48.899	47.6134
RD	θ = 1.5213	0.1523	29.8533	61.7066	62.7023	61.9288
IR	θ = 2.8184	0.356	36.1725	74.345	75.3407	74.345

Tables 3 and 4 shows the 95% confidence interval constructed for the parameters of the IPR distribution, using the first and second datasets respectively.

Table 3: MLEs of the parameters IPR distribution and their C.I

Model	MLE	S.E	95% Confidence Interval	
			Lower Limit	Upper Limit
IPR	α = 2.2466	0.2225	1.8105	2.6827
	θ = 144.6290	67.4959	12.337	276.921
RD	θ = 0.2956	0.0192	0.2589	0.3332
TPPR	α = 0.9463	0.0792	0.7911	1.1015
	θ = 0.3419	0.0763	0.1924	0.4914
IR	θ = 10.5177	1.3291	7.9127	13.1227

Table 4: MLEs of the parameters IPR distribution and their C.I

Model	MLE	S.E	95% Confidence Interval	
			Lower Limit	Upper Limit
IPR	α = 4.1181	0.6671	2.8106	5.4256
	θ = 6.6111	1.836	3.0125	10.201
TPPR	α = 1.7828	0.1972	1.4448	2.1694
	θ = 1.0341	0.1624	7.16E-01	1.3524
RD	θ = 1.5213	0.1523	1.2228	1.8199
IR	θ = 2.8184	0.356	2.1207	3.5161

Results in Tables 3 and 4 shows that the parameter estimates of the IPR distribution lies within the confidence limits.

6 Conclusions

In distribution theory, we often make effort to generalize a distribution. The essence of generalization is to improve the specific distribution under consideration and to make it more flexible so as to extend its application to other areas. The superiority of pragmatic outcomes obtained from any distribution by employing parametric techniques lays on the capability of the data to fit appropriately, the distribution under consideration. In this paper, we have proposed a new distribution known as the Inverted Power Rama distribution. The mathematical characteristics and reliability measures such as survival function, hazard rate and odds function are derived. The method of maximum likelihood was used to estimate the parameters of the distribution and however, the distribution was subjected to real life data to illustrate its application. Based on the empirical results obtained, the IPR distribution outperforms the competing models considered in the article. Hence, we recommend the used of the proposed model when modeling lifetime data that are heavy tailed and has upside down bathtub shape.

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