THEORETICAL AND EXPERIMENTAL STUDY OF THE DISTRIBUTION OF STRESS WAVES IN LINEAR VISCOELASTIC MEDIA

Abstract.
The influence of viscoelastic properties of an optically sensitive epoxy material on the propagation of pressure waves in a rod was carried out. In the article of determination of the visco-elastic operator, the theoretical relations for the case of propagation of longitudinal waves are obtained on the basis of the results of a photoelastics experiment. The noticeable difference between the experimental data and the calculation at the pulse end can be explained by the change in the propagation velocity of the Fourier decomposition component of the real impulse.

Keywords: photoelastics experiment, viscoelastic properties, Fourier decomposition, waves of pressure.

NOMENCLATURE

$\ell =$ density	$\delta_{ij} =$ stress tensor
$\lambda, \mu =$ elastic constant	$\varepsilon_{ij} =$ strain tensor
$\alpha, c =$ attenuation coefficient	$\mathrm{u} =$ displacement
$\mathrm{L(h), M(h)} =$ linear integral operators	$\mathrm{f_1, f_2} =$ viscoelastic operators
$\mathrm{A(p)} =$ p - circular frequency	$\mathrm{h_1} =$ spectral continous function
$\mathrm{h_2} =$ spectral discrete function	

1. Introduction

In this paper, theoretical data for the case of propagation of one-dimensional longitudinal waves are obtained for determining viscoelastic operators from experimental data. A review of publications on the use of the polarization-optical method for studying stress waves in isotropic [1] and anisotropic [2] geophysical media suggests that one of the main issues is the display of viscous properties by polymeric optically-responsive materials [3]. A number of authors obtained materials for photoelastic modeling in which the viscoelastic properties were minimized and this allowed them to be used in modeling two-dimensional dynamic problems of mechanics of anisotropic bodies [2]. However, in most cases it becomes necessary to study and evaluate the viscoelastic properties of such materials and to take them into account in the dependencies connecting optical and mechanical quantities. The behavior of many polymers can be described with sufficient accuracy using linear differential or integral operators. The definition of such operators in the general case is a rather complicated task [4].

The influence of the viscoelastic properties of the optically-responsive material ED20-THFA on the propagation of a pressure wave in the rod was analyzed.

2. STATEMENT OF THE PROBLEM

To describe the process of deformation of a viscoelastic medium, we introduce a coordinate frame:

The equation of motion of a viscoelastic medium in coordinates x_j (j = 1,2,3) has the form:

$$\sigma_{kl,l} = \rho \frac{\partial^2 u_k}{\partial t^2} (k, l = 1,2,3) \qquad (1)$$

The relationship between the components of the stress tensor σ_{ij} and the strain tensor ε_{ij} will be given in the form of the Boltzmann relations

$$\left.\begin{aligned} \sigma_{ij} &= L(\varepsilon) + 2M(\varepsilon_{ij})(j = 1,2,3); \\ \sigma_{ij} &= M(\varepsilon_{ij})(i \neq j;\ i, j = 1,2,3). \end{aligned}\right\} \qquad (2)$$

Here L (h) and M (h) are linear integral operators of the type:

$$\left.\begin{aligned} L\,(h) &= \lambda h - \int_{-\infty}^{t} f_1(t-\xi) h(\xi) d\xi; \\ M\,(h) &= \mu h - \int_{-\infty}^{t} f_2(t-\xi) h(\xi) d\xi, \end{aligned}\right\} \qquad (3)$$

where λ, μ are elastic constants; f_1 (t), f_2 (t) -core of viscoelastic operators, having the following form:

$$\left.\begin{aligned} f_1\,(t) &= \lambda \int_0^{\infty} \frac{h_1(\xi)}{\xi^2} exp\left(-\frac{t}{\xi}\right) d\xi + \lambda \sum_{k=1}^{n} \frac{\gamma_k}{\tau_k} exp\left(-\frac{t}{\tau_k}\right) \\ f_2\,(t) &= \mu \int_0^{\infty} \frac{h_2(\xi)}{\xi^2} exp\left(-\frac{t}{\xi}\right) d\xi + \mu \sum_{k=1}^{n} \frac{\beta_k}{\tau_k} exp\left(-\frac{t}{\tau_k}\right), \end{aligned}\right\} \qquad (4)$$

where h_1 (ξ), h_2 (ξ), γ_k, β_k are spectral continuous and discrete functions of relaxation times ξ and τ_k, respectively, with h_1 (ξ) and h_2 (ξ) bounded on the half-interval $0 \leq \xi < \infty$ and $\xi \approx 0$.

$$h_1(\xi) = 0(\xi^{1+\alpha}), h_2(\xi) = 0(\xi^{1+\alpha}), \alpha > 0 \qquad (5)$$

We will assume that $\vec{u} = grad\Phi + rot\vec{\psi}, \vec{\psi}(0, \psi_1\psi_2),$ (6)

where $\vec{u}$ is the displacement vector (u_1, u_2, u_3).

The equations of motion (1) when performing relations (2) are reduced to the form

$$\Delta\Phi - \frac{1}{\lambda + 2\mu}\int_{-\infty}^{t}[f_1(t-\xi) + 2f_2((t-\xi)]\,\Delta\Phi d\xi = \frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2}, a^2 = \frac{\lambda + 2\mu}{\rho}; \quad (7)$$

$$\Delta\vec{\psi} - \frac{1}{\mu}\int_{-\infty}^{t} f_2(t-\xi)\,\Delta\vec{\psi}d\xi = \frac{1}{b^2}\frac{\partial^2\vec{\psi}}{\partial t^2},\ b^2 = \frac{\mu}{\rho}, \quad (8)$$

where Δ is the three-dimensional Laplace operator.

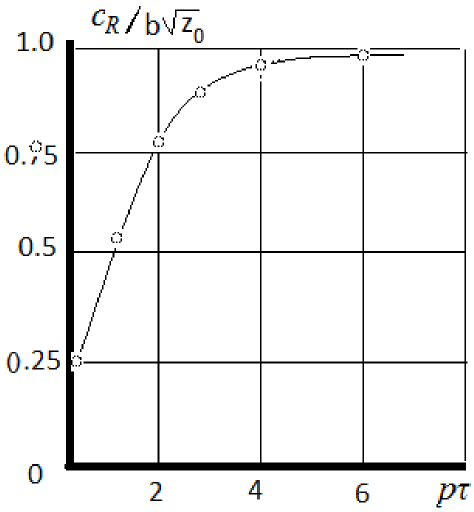
Note that for an isotropic viscoelastic medium, the operators *L* (h) and *M* (h) must satisfy the inequalities

$$\Delta\vec{\psi} - \frac{1}{\mu}\int_{-\infty}^{t} f_2(t-\xi)\,\Delta\vec{\psi}d\xi = \frac{1}{b^2}\frac{\partial^2\vec{\psi}}{\partial t^2},\ b^2 = \frac{\mu}{\rho}, \quad (9)$$

where L_0 and M_0 are the Fourier transformed operators *L* and *M*.

As we see, studies of dynamic problems for linear viscoelastic media are narrowed to solving integro-differential equations (7) - (8) under given initial and boundary conditions. If a viscoelastic medium occupies a half-space $y \leq 0$, then a flat deformed or generalized plane stress state takes place, and the wave field does not depend on one of the coordinates, for example, on *z*, and instead of the vector ψ in equation (8) it is necessary to substitute the potential function (x , y).

Consider a problem, the results of which can be applied to the definition of viscoelastic operators according to experimental data.



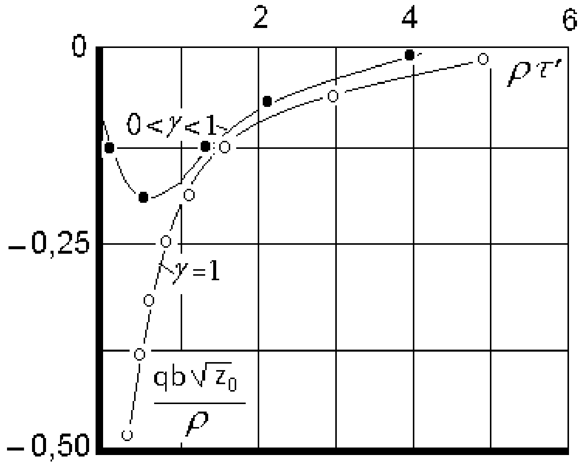


Fig.1. Fig.2.

From curves 1 and 2, it follows that for waves that do not contain high harmonics, the dependence of the parameters of the medium on the constants determining the viscoelastic medium is insignificant and the viscoelastic body behaves like an elastic isotropic body. The dependence of c_R and q_2 is studied in a similar way in a more general case.

From the solution of the one-dimensional wave problem for an elastic isotropic medium, it follows that the longitudinal stress σ_x is determined by the formula

$$\sigma_x(x,t) = -\rho c\dot{u}(x,t), \qquad (10)$$

where ρ is the density of the material, which can be considered constant and under dynamic loading conditions; c is the velocity of a longitudinal wave in an elastic unbounded medium or the velocity of propagation of an elastic wave in a rod; $\dot{u}$ is the speed of particles behind the front of an elastic wave.

In the case of a one-dimensional problem for a linear viscoelastic medium, using the correspondence principle, instead of (10) we will have

$$\sigma_x^*(x,t) = -\rho \int_0^t c(t-\tau)d\dot{u}(x,\tau) \qquad (11)$$

Substituting expressions (11) into the equation of motion (1) with $\tau\ (x, y) = 0$, we get:

$$-\int_0^t c(t-\tau)d\acute{\varepsilon}(x,\tau) = \ddot{u}\,(x,t) \qquad (12)$$

We integrate (12) over time t using zero initial data:

$$-\int_{0}^{t} c(t-\tau)\varepsilon(x,\tau)d\tau = u\,(x,t) \qquad (13)$$

By approximating ε (x, t) and u (x, t) experimentally measured in rods of viscoelastic materials, it is possible to determine from equation (13) and, therefore, to establish a relationship between viscoelastic operators. Relationship (13) are sufficient to determine the operators connecting stresses and strains in a linear viscoelastic medium, according to experimentally measured values.

3. RESULTS

Having performed integration of the parts in the expression (11) with allowance for zero initial conditions, we obtain the formula for determining stresses in a viscoelastic medium

$$\sigma_x^*(x,t) = -\rho\left[c(0)\dot{u}(x,t) + c'(0)u(x,t) + \int_{0}^{t} c''\,(t-\tau)u(x,\tau)d\tau\right], \qquad (14)$$

where c (0) is the velocity of propagation of the disturbance front, c ^ (0) is the time derivative of the velocity of propagation of the disturbance on the wave front, having the dimension of acceleration.

The first member of formula (10) is similar to the solution of an elastic problem, and the subsequent terms are characteristic only of materials with viscous resistance. If the right side of the expression (10) is limited to the first two terms, then we obtain the formula for the approximate determination of the voltages in the pulse. For environments with little internal absorption, this approximation is likely to be acceptable.

Suppose that at time t_i, the stress σ_x ^ * (x, t) reaches a maximum in the section x_n under study. Then the derivative of σ_x ^ * (x, t) with respect to x should be equal to zero. Take the derivative with respect to x from the first two terms of expression (14):

$$\frac{\partial\sigma_x^*(x,t)}{\partial x} = -\rho\left[c(0)\frac{\partial^2 u}{\partial x\partial t} + c'(0)\frac{\partial u}{\partial x}\right], \qquad (15)$$

or, given that $\partial u/\partial x = \varepsilon_x$,

$$\frac{\partial \sigma_x^*(x,t)}{\partial x} = -\rho \left[c(0) \frac{\partial \varepsilon_x}{\partial t} + c'(0)\varepsilon_x \right] \qquad (16)$$

Equating expression (16) to zero, we obtain the formula for determining c '(0):

$$c'(0) = -c(0) \frac{\partial \varepsilon_x / \partial t}{\varepsilon_x} \qquad (17)$$

Substituting (17) into (15), we obtain the expression for determining the stresses in a rod of a linear viscoelastic material during the propagation of a longitudinal pressure pulse

$$\sigma_x^*(x,t) = -\rho c(0) \left[\frac{\partial u}{\partial x} - u \frac{\partial \varepsilon_x / \partial t}{\varepsilon_x} \right] \qquad (18)$$

To determine stresses by formula (18), it is necessary to know the position of the σ and ε curves at the determined time point. In the general case, for a viscoelastic material, the stress and strain curves under the action of a dynamic load are shifted in phase, and it is impossible to determine by direct methods the magnitude of this shift for a pulse. Indirectly, the position of the curve ε relative to σ can be estimated based on frequency tests to determine the tangent of the angle of mechanical loss tgγ [3]. The table shows the results of stress calculations from dependence (14) for different shear values of the ε and σ curves for the ED20 – TGFA material.

The values of ε_x and $(\partial \varepsilon_x)/\partial t$ during the propagation of the longitudinal pressure pulse in the rod were determined using the graphical differentiation of the experimental curves u (x, t).

According to the results of tests using the resonance method, it was obtained that the tangent of the angle of mechanical losses for a material ED20-THFA (epoxy systems) does not depend on the frequency and is approximately 0.01, which corresponds to $\gamma \approx 1^0$. As can be seen from the table, for $\gamma = 0.9^0$, the second term of formula (16) is only 3.6% of the first and the stresses in such a material can be determined from the simplified dependence (10).

γ, град	u,м	$\dot{u}$, м/с	ε	$\dot{\varepsilon}$, с$^{-1}$	$u\frac{\dot{\varepsilon}}{\varepsilon}$, м/с	$\rho c \dot{u}$, Н/м^2	$\rho c \dot{u}$, Н/м^2	$\frac{\rho c' u}{\rho c \dot{u}} \times 100$
9,00	$1{,}21 \cdot 10^{-4}$	2800	$1{,}47 \cdot 10^{-2}$	$1{,}16 \cdot 10^{3}$	5,00	$6{,}376 \cdot 10^{7}$	$2{,}168 \cdot 10^{7}$	34,0
6,75	$1{,}29 \cdot 10^{-4}$	2850	$1{,}50 \cdot 10^{-2}$	$0{,}82 \cdot 10^{3}$	3,73	$6{,}494 \cdot 10^{7}$	$1{,}619 \cdot 10^{7}$	25,0

4,50	$1{,}40\cdot10^{-4}$	2900	$1{,}52\cdot10^{-2}$	$0{,}58\cdot10^{3}$	2,79	$6{,}612\cdot10^{7}$	$1{,}206\cdot10^{7}$	18,0
2,25	$1{,}50\cdot10^{-4}$	2940	$1{,}55\cdot10^{-2}$	$0{,}37\cdot10^{3}$	1,90	$6{,}700\cdot10^{7}$	$0{,}824\cdot10^{7}$	12,0
0,90	$1{,}57\cdot10^{-4}$	2950	$1{,}55\cdot10^{-2}$	$0{,}10\cdot10^{3}$	0,56	$6{,}729\cdot10^{7}$	$0{,}245\cdot10^{7}$	3,6

When studying the propagation of waves of arbitrary shape in viscoelastic materials and studying their dynamic properties, the application of Fourier analysis [5] may be of considerable interest. By decomposing the applied impulse of a given shape into a Fourier series, it is possible to determine the shape of the impulse that has spread to any distance, applying the superposition principle for viscoelastic media.

For a flat pulse applied to the end of the rod, the voltage can be expressed as a Fourier integral

$$\sigma(0,t) = \int_0^\infty A(p)e^{i\rho t}dp, \qquad (19)$$

where *A (p)* generally has a complex form; p-circular frequency. Then the stress *σ (x, t)*, caused by the pulse at a distance x from the source, can be expressed as

$$\sigma(x,t) = \int_0^\infty A(p)e^{ipt-\left(\propto+\frac{ip}{c}\right)x}dp, \qquad (20)$$

Where ∝ and *c* are the attenuation coefficient and the phase velocity of the wave, which are generally functions of *p*.

The propagation of a longitudinal pressure pulse in a rod made of material ED20-THF was studied by the polarization-optical method. The pulse was initiated by the explosion of the lead azide charge at the end of a square rod (4 × 4 mm) (Fig. 3).

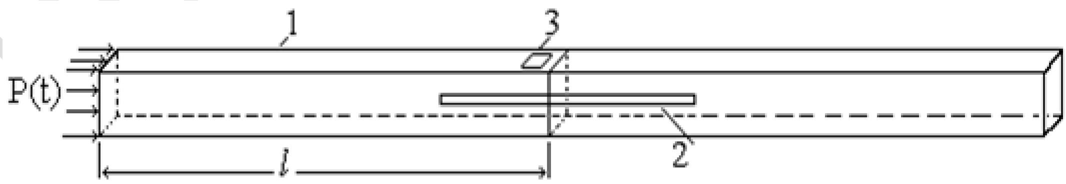


Fig. 3

Stress waves were fixed using a polarization-dynamic setup [6] in the photo recorder mode in the form of interference patterns of strips at different distances along the rod (Fig. 4). The velocity of propagation of the wave front and the pulse components measured by kilograms in the investigated distance range (up to 400 mm) varies within 3–4% and can be regarded as constant in the first approximation in formula (16).

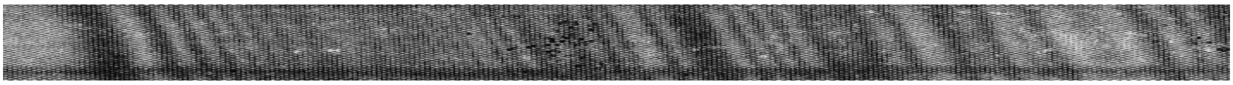


Fig. 4

The impulse σ_x (0, t) initiated at the rod end is well approximated by the expression

$$\sigma_x(0,t) = \sigma_0 sin^4 \frac{\pi t}{T}, \qquad (21)$$

where σ_0 is the amplitude; T-pulse duration.

Applying the inverse Fourier transform to the expression (19) and taking into account (21), we can determine the distribution of amplitudes, harmonic components of the pulse frequency

$$A(p) = \sigma_0 \int_0^T sin^4\left(\frac{\pi t}{T}\right) e^{-ipt} dt \qquad (22)$$

Fig. 5 shows the dependence of the amplitude on the frequency, from the analysis which it follows, that in the considered pulse the main carrier frequencies lie in the range from 50 to 4000 Hz.

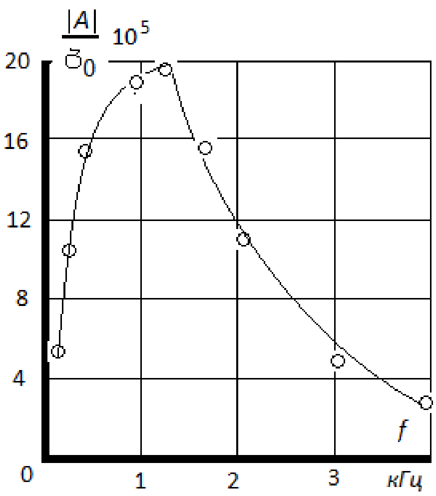


Fig 5.

Considering that the wave speed in the rod made of material ED20-THF is almost constant, the main influence of viscoelastic properties is manifested in the attenuation of the pulse amplitude with increasing distance, which is taken into account by the attenuation coefficient ∝, which we take in the form

$$\propto \approx \propto_0 + \frac{\propto_1}{ip}, \qquad (23)$$

Where $\propto_0$ and $\propto_0$ are constant coefficients.

We use the convolution theorem and transform expression (20) to the form

$$\frac{\sigma(x,t)}{\sigma_0} = e^{-\alpha_0 x}\left\{ sin^4 \frac{\pi}{T}\left(t - \frac{x}{c}\right) - 2\sqrt{\alpha_1 x}\int_0^{\sqrt{t-\frac{x}{c}}} sin^4\left[\frac{\pi}{T}\left(t - \frac{x}{c} - \tau^2\right)\right] I_1\left(2\tau\sqrt{\alpha_1 x}\right) d\tau \right\}, \qquad (24)$$

where I_1 is the Bessel function of the 1st kind; τ is a variable integration.

The experimental pulse shape was compared with the shape obtained on the basis of Fourier analysis for different values of α_0 and α_1.

For example, Fig. 6 shows the graphs $(\sigma_x(t)) / \sigma_0$ at a distance of 200 mm from the rod end.

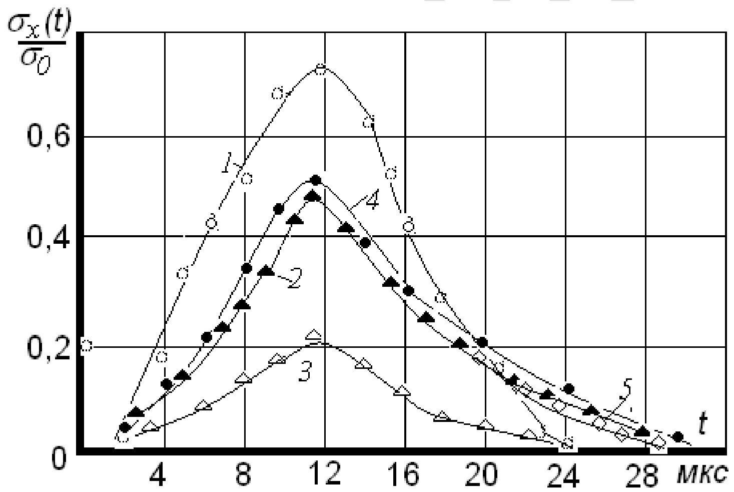


Fig.6.

Good agreement between the experimental data and the calculation according to (24) takes place at = 0, 003 and at = 0.0002 ÷ 0.001. The noticeable discrepancy between the experimental data and the calculation at the end of the pulse is apparently due to some change in the propagation velocity of the components of the Fourier decomposition of the real pulse.

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