Large deviation principle for stochastic differential system pertubated by a rapid process in the Besov-Orlicz topology

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Résumé

Dans cet article, nous étudions les résultats de grandes déviations d'une famille de processus X^ε perturbé par un processus rapide ζ dans l'espace de Besov-Orlicz. Le processus X^ε est une solution de l'EDS interprétée au sens d'Itô :

$$\begin{cases} dX_t^\varepsilon & = b(X_t^\varepsilon, \zeta_{t/\varepsilon})\ dt + \sqrt{\varepsilon}\sigma(X_t^\varepsilon)\ dW_t \\ X_0 & = x \in \mathbb{R}^d \end{cases}$$

où ζ est un processus indépendant du mouvement brownien W et satisfait un principe de grandes déviations.

Mots clés :Principe de grandes déviations, principe de moyennisation, espace de Besov-Orlicz

Abstract

In this study, we are going to see a large deviation principle associated with a family process X^ε perturbated by a rapid process ζ in the Besov-Orlicz space. The process X^ε is a solution of Itô integral :

$$\begin{cases} dX_t^\varepsilon & = b(X_t^\varepsilon, \zeta_{t/\varepsilon})\ dt + \sqrt{\varepsilon}\ \sigma(X_t^\varepsilon)\ dW_t \\ X_0 & = x \in \mathbb{R}^d \end{cases}$$

in which the condition ζ is independent of the brownian motion W and obeys a large deviation principle.

Key words :Large deviation, averaging principle, Besov-Orlicz space

1 Introduction

In this study, we consider a diffusion processes X^ϵ d-dimensional solution of stochastic differential equation (SDE) :

$$dX_t^\varepsilon = b(X_t^\epsilon, \zeta_{t/\varepsilon})\ dt + \sqrt{\varepsilon}\ \sigma(X_t^\varepsilon)\ dW_t, \quad X_0 = x \in \mathbb{R}^d \tag{1}$$

where W is a Wiener's standard process independent of ζ. Our purpose here is to etablish the asymptotic evaluation of $P(X_t^\epsilon \in A)$ where A is a Borel set of Besov-Orlicz space under the assumption that the process X_t^ϵ converges to the solution $\bar{X}_t$ defined by :

$$\begin{aligned} d\bar{X}_t &= \bar{b}(\bar{X}_t)\ \ dt, \quad \bar{X}_0 = 0, \\ \bar{b}(x) &= \lim_{T\to\infty} \frac{1}{T}\int_0^T b(x_s, \zeta_{s/\epsilon})\ ds \end{aligned}$$

The asymptotic evaluation obtained will be the result of a large deviations from X_t^ϵ compared to $\bar{X}_t$.

The basic work on the subject is the article by Freidlin [10], se also refered to Ventcel's book - Freidlin [9]where he gets this evaluation under the assumption :

$$\lim_{T\to\infty} \frac{1}{T} \log E\left(\exp(\int_0^T \left\langle \alpha, b(x_s, \zeta_{s/\epsilon})\right\rangle)\ ds\right) = H^0(x, \alpha) \tag{2}$$

exists uniformly in x and differentiable in α.

The special case $\zeta \equiv 0$ $(b(X_t^\epsilon, 0) = b(X_t^\epsilon))$ and $\sigma \not\equiv Id$ was studied by Freidlin & Wentzell [8] se also refered to Varadhan [27], Azencott [1] and Stroock [26] with the usual topology uniform, Ben Arous and Ledoux [5] have developed a large deviation principle(LDP) in Hölder's space. Later on, an extension to Besov's space was considered in Eddahbi et al [7] and Roynette's [4]

The particular case $\sigma \equiv 0$, $\zeta \not\equiv 0$ and $b \not\equiv Id$ have been studied by M. BRANCOVAN [3]. The case $\zeta \not\equiv 0$ was studied by A. GUILLIN [13] in a moderate deviation situation.

The aim of this paper is to study the large deviation principle (LDP) of the law of $\{X_t^\epsilon, \epsilon > 0\}$ in the Besov-Orlicz topology. This is the extension of the result of H. LAPEYRE [17] in a stronger topology.

The paper is organized as follows : In section 2, we introduce some hypotheses and notations. Section 3 contains some preliminary definitions and general results which are essential for the proof of the theorem (4.4). Section 4, under the hypotheses in section 2, we prove in theorem (4.1) the LDP of X_t^ϵ, solution of (1) when ζ satisfies a large deviation principle.

2 Hypotheses and Notations

2.1 Hypotheses

In this paper, we assume that the following hypotheses will be verified :

H1. The function $\sigma : \mathbb{R}^l \times \mathbb{R}^d \to \mathbb{R}^d \times \mathbb{R}^r$ is jointly measurable in (x, y) and there exists a constant $C > 0$ such that.

$$\begin{aligned} &|\sigma(x,y) - \sigma(x',y')| \leqslant C(|y-y'| + |x-x'|) \\ &|\sigma(x,y)| \leqslant C \end{aligned}$$

H2. The function $b : \mathbb{R}^l \times \mathbb{R}^d \longrightarrow \mathbb{R}^d$ is jointly measurable in (x, y) and there exists a constant $C > 0$ such that.

$$\begin{aligned} &|b(x,y) - b(x',y')| \leqslant C(|y-y'| + |x-x'|) \\ &|b(x,y)| \leqslant C|x-y| \end{aligned}$$

H3. W is a standard $\mathbb{R}^r$-valued Brownian motion

H4. $\zeta_{t/\epsilon}$ is a process R^l-value independent of brownian motion W and obeys a large deviation principle with a good rate function I.

2.2 Notations

2.2.1 Cameron-Martin space

Let $H(\mathbb{R}^d)$ be the Cameron-Martin space associated with the Brownian motion on $\mathbb{R}^d$

$$H(\mathbb{R}^d) = \left\{ \begin{array}{c} f : [0,1] \rightarrow \mathbb{R}^d, f \text{ is absolutely continuous such that} \\ f(0) = 0 \text{ and } \displaystyle\int_0^1 |\dot{f}_s|^2 \; ds < +\infty \end{array} \right\}$$

$H(\mathbb{R}^d)$ is a Hilbert Space equipped with the norm

$$\langle f, g \rangle = \int_0^1 \dot{f}_s \dot{g}_s \; ds$$

2.2.2 Besov-Orlicz space

Let B_{M_β, w_α} be denote the Besov-Orlicz space of continuous function $f : [0,1] \rightarrow \mathbb{R}^d$ such that $\parallel f \parallel_{M_\beta, w_\alpha} < \infty$. For all $\alpha > 0$, let us put

$$\parallel f \parallel_{M_\beta, w_\alpha,} = \parallel f \parallel_{M_\beta} + \sup_{0 \leq t \leq 1} \frac{w_{M_\beta}(f,t)}{w_{\alpha,\lambda}(t)}$$

where $w_{\alpha,\lambda}(t) = t^\alpha \left(1 + \log \frac{1}{t}\right)^\lambda, \forall \alpha > 0, \parallel f \parallel_{M_\beta} = \inf \left\{ \tau > 0, \frac{1}{\tau} \left[1 + \int_0^1 M_\beta(\tau |f(t)|) dt \right] \right\}$ et $w_{M_\beta}(f,t) = \sup\limits_{0 \leq h \leq t} \parallel \Delta_h f \parallel_{M_\beta}$ with

$$\Delta_h f(x) = 1_{[0,1-h]}(x)(f(x+h) - f(x)), \forall h \in [0,1].$$

We will use the equivalent of Cieleski, Z. [4]. Let $\chi_1, \chi_{j,k}, j = 0, 1 ..., k = 1 ... 2^j, supp\chi_{j,k} = [(k-1)/2^j, k/2^j]$, be the set of Haar functions over the interval $[0,1]$, and let $\varphi_0(t) = 1, \varphi_1(t) = t, \varphi_{j,k}(t) = \int_0^t \chi_{j,k}(s) ds$ be the set of Schauder functions. Let $f : [0,1] \rightarrow \mathbb{R}^d$ be a continuous functions , let us note by $\{A_n(f), n \geq 0\}$ the coefficients of the decomposition of f in the Schauder basis given by

$$f(t) = A_0(f)\varphi_0(t) + A_1(f)\varphi_1(t) + \sum_{n=2^j+1}^{2^{j+1}} \sum_{j,k} A_n(f) \varphi_{j,k}(t)$$

where $A_0(f) = f(0), A_1(f) = f(1) - f(0)$ and

$$A_n(f) = 2^{\frac{j}{2}} \left[\left(f\Big(\frac{2k-1}{2^{j+1}}\Big) - f\Big(\frac{2k-2}{2^{j+1}}\Big) \right) - \left(f\Big(\frac{2k}{2^{j+1}}\Big) - f\Big(\frac{2k-1}{2^{j+1}}\Big) \right) \right]$$

Let $B^0_{M_\beta,w_\alpha}$ be the subspace of B_{M_β,w_α} corresponding to the sequence $f_{j,k}$ such that

$$B^0_{M_\beta,w_\alpha} = \Big\{ f \in \mathcal{C}([0,1],\mathbb{R}^d);\, \| \, f \, \|_{M_2,w_\alpha} < \infty,\, \lim_{j\wedge p\longrightarrow\infty} 2^{-j(\frac{1}{2}-\alpha+\frac{1}{p})} p^{-\gamma}(1+j)^{-\lambda} \parallel f_{j,.} \parallel_p = 0 \Big\}$$

where

$$\parallel f_{j,.} \parallel_p = \Big(\sum_{k=1}^{2^j} |f_{j,k}|^p \Big)^{\frac{1}{p}} \textit{ and } \beta\gamma = 1$$

$B^0_{M_\beta,w_\alpha}$ is a Banach space.

For more details on Besov-Orlicz space we refer to instance [4].

3 Preliminary definitions and results

3.1 Preliminary definitions

Definition 3.1. A mapping $I : E \longrightarrow [0;+\infty]$ is said to be a rate function if it is lower semicontinuous (lsc). Furthermore, we will say that I is a good rate function, if for any $a < +\infty$, the set $\Gamma_a = \{x \in E, I(x) \leq a\}$ is compact.

Unless explicitly stated otherwise, for any subset A of E and any rate function, we note $I(A) = \inf_{x\in A} I(x)$.

Definition 3.2. For some mapping I, the family of probabilities $\{P^\varepsilon\}_{\varepsilon>0}$ satisfies a large deviation principle (LDP) if the following holds :

i) (Lower bound.) For every open subset O of E

$$\liminf_{\varepsilon\longrightarrow 0} \varepsilon \log P_\varepsilon(O) \geq -I(O)$$

ii) (Upper bound.) For every closed subset F of E

$$\limsup_{\varepsilon\longrightarrow 0} \varepsilon \log P_\varepsilon(F) \leq -I(F).$$

3.2 Preliminary results

We will use the following characterization theorem.

Theorem 3.3. *Let $p_0 \geq 1$, f belongs to $B^0_{M_\beta,w_\alpha}$ if and only if*

$$\max\Big(|f_0|, |f_1|, \sup_{p\geq p_0} \sup_{j\geq 0} 2^{-j(\frac{1}{2}-\alpha+\frac{1}{p})} p^{-\gamma} (1+j)^{-\lambda} \parallel f_{j,.} \parallel_p \Big) < \infty \tag{3}$$

Theorem 3.4. *Let f belongs to $B^0_{M_\beta,w_\alpha}$ if and only if*

$$\lim_{j\vee p\to p_0} 2^{-j(\frac{1}{2}-\alpha+\frac{1}{p})}p^{-\gamma}(1+j)^{-\lambda} \parallel f_{j,.} \parallel_p \Big) < \infty \tag{4}$$

For the proof of this result we refer to [4]

Consider the following norm which is are crucial to prove our results :

$$\parallel f \parallel_{**} = \sup_{0\leq s<t\leq 1} \frac{|f(t)-f(s)|}{w(t-s)}$$

this is dominated by

$$\parallel f \parallel_{*} = \max\Big(|f(1)|, \sup_{j\geq 0}\ \sup_{0\leq k\ \leq 2^j} \frac{|f_{j,k}|}{\sqrt{1+j}}\Big).$$

It is easy to show that there exist $D_1 > 0$ and $D_2 > 0$ such that
$\parallel f \parallel_{M_2,w} \leq D_1 \parallel f \parallel_{**} \leq D_2 \parallel f \parallel_{*}$.

The following LDP proved by Baldi et al. (1992) extends the classical Schilder theorem (see Schilder 1996 ; Deuschel and Strook 1989)

Theorem 3.5. *Let P^ε be the law of $\sqrt{\varepsilon}W$ on $B^0_{M_2,w_\alpha}$ equipped with the norm $\|\|_{M_\beta,w_\alpha}$ satisfying the LDP with the good rate function $I(.)$ defined by :*

$$I(f) = \begin{cases} \dfrac{1}{2}\displaystyle\int_0^T |\dot{f}(s)|^2\, ds & \textit{if } f\in H(\mathbb{R}^d) \\ +\infty & \textit{otherwise} \end{cases}$$

One of the basic tools in large deviation theory is the 'contraction principle' (see Deuschel and Strook 1989). It enables the new rate function to be computed after the data have been transformed by a continuous map [18].

Theorem 3.6. *Let Q^ε be a family of probability measure on a Polish space E and satisfies the LDP with a good rate function λ.*
Let $F : E \longrightarrow E'$ be countinuous. Denote by $Q^\varepsilon = P^\varepsilon \circ F^{-1}$ the family of image measure of P^ε, then $\{Q^\varepsilon\}$ satisfies the LDP with a good rate function $\tilde{\lambda}$ defined by

$$\tilde{\lambda}(y) = \inf_{x:f(x)=y} \lambda(x).$$

Lemma 3.7. *There exist $C = C_l$ such that for all $\lambda > 0$ and $\mu > 0$ where $\lambda > 4l\mu > 0$ and $\lambda > 2\sqrt{\log 2}$, we have*

$$P\Big[\parallel W \parallel_{**} \geq \lambda, \parallel W \parallel \leq \mu\Big] \leq C\max\Big(1, l\Big(\frac{\lambda}{4l\mu}\Big)^2 \exp\Big(-\frac{\lambda^2}{C}\ln\Big(\frac{\lambda}{4l\mu}\Big)\Big)\Big) \tag{5}$$

Lemma 3.8. *(Exponential inequality)*
For all $u > 2\sqrt{\log 2}$ *and for all process* K *on* $[0,1]$ *there exist* $C = C_l$ *such that*

$$P\Big[\parallel \int_0^{\cdot} K_s \; dW_s \parallel_{**} \geq u, \parallel K \parallel \leq 1 \Big] \leq C \exp\Big(-\frac{u^2}{C} \Big). \tag{6}$$

Now, we give a new formulation for the contraction principle which will be needed later.

Lemma 3.9. *Let* $(E_X, d_X), (E_Y, d_Y), (E_Z, d_Z), (E, d)$ *denote Polish spaces and* (Ω, F, P) *be a probability space.*

Suppose that $(X^\varepsilon, \varepsilon > 0)$ *is a family of random variables with values in* E_X *satisfing a LDP with a rate function* I_X*, and* $(Y^\varepsilon, \varepsilon > 0)$ *a random variable with values in* E_Y *satisfing a LDP with a rate function* I_Y*.*

Suppose that for each $\varepsilon > 0$*,* X^ε *is independent of* Y^ε *then the family of random variable* $Z = F(X^\varepsilon, Y^\varepsilon)$ *satisfing a LDP with rate function* $I_F(z)$ *defined by*

$$I_F(z) = \inf_{F(x,y)=z} I_X(x) + I_Y(y).$$

where $F : E_X \times E_Y \rightarrow E_Z$ *is continuous.*

The main purpose of the following section is to build a functional controlling the large deviation of X^ε on $B^0_{M_2, w_\alpha}$ if we know the large deviation of ζ in $B^0_{M_2, w_\alpha}$. More precisely, we are building for all $T > 0$ a functional S_T satisfying the following assertions :

i) For each positive α, $K_\alpha = \{\varphi \in B^0_{M_2, w_\alpha} / S_T(\varphi) \leq \alpha\}$ is a set compact

ii) For every open subset O of $B^0_{M_2, w_\alpha}$,

$$\overline{\lim_{\varepsilon \to 0}} \, \varepsilon \log P(X^\varepsilon(x) \in O) \geq -\inf_{\varphi \in O} S_T(\varphi).$$

iii) For every closed F of $B^0_{M_2, w_\alpha}$,

$$\overline{\lim_{\varepsilon \to 0}} \, \varepsilon \log P(X^\varepsilon(x) \in F) \leq -\inf_{\varphi \in F} S_T(\varphi).$$

where

$$S_T(\varphi) = \inf\{s^\varphi(g, f), \psi = F^\varphi(g, f)\}.$$

The aim of this study is to establish the large deviation principle of X^ε in $B^0_{M_2, w_\alpha}$ by using the Azenctott's method in a general setting. As a reminder for Azenctott's method , let (E_i, d_i), $i = 1, 2$ be two Polish spaces and $X^i_\varepsilon \rightarrow E_i$, $\varepsilon > 0, i = 1, 2$ two families of random variables. Assume that $\{X^\varepsilon_1 \,, \varepsilon > 0\}$ satisfies a LDP with rate function $I_1 : E_1 \rightarrow [0, +\infty]$. Let $\Phi : \{I_1 < \infty\} \rightarrow E_2$ be a mapping such that its restriction to the compact sets $\{I_1 \leq a\}$ is continuous in the topology of E_1. For any $g \in E_2$ we set $I(g) = Inf\{I_1(f), \Phi(f) = g\}$. Suppose that for $R, \rho, a > 0$ there exist α and $\varepsilon_0 > 0$ such that for $f \in E_1$ satisfying $I_1(f) \leq a$ and $\varepsilon \leq \varepsilon_0$ we have

$$P\{d_2(X_2^\varepsilon, \Phi(f)) \geq \rho, d_1(X_1^\varepsilon, f) \leq \alpha\} \leq exp(\frac{-R}{\varepsilon^2}) \tag{7}$$

Then the family $\{X_2, \varepsilon > 0\}$ satisfies a LDP with rate function I.

4 The main result

Theorem 4.1. *Assume that H_1, H_2. Let X^ε is the unique solution of* (1). *Then the family $\{X^\varepsilon\}_{\varepsilon>0}$ satisfies a LDP in $B^0_{M_2,w_\alpha}$ with a good rate function defined by*

$$I(f) = \begin{cases} \frac{1}{2}\int_0^T |\dot{h}(s)|^2 \, ds & \text{if } h \in H(\mathbb{R}^d), f = S(h) \\ +\infty & \text{otherwise} \end{cases}$$

where S(h) is the unique continous solution of

$$dS_t(h) = b(S(h)(t), \xi_{t/\varepsilon}) \, dt + \sqrt{\varepsilon}\sigma(S(h)(t)) \, \dot{h}(t) \, dt$$

For the proof of the Theorem (4.1), we will be interested in the behavior of X_ε in a tube around a function φ absolutely continuous in $\Big(\mathcal{C}([0,1], \mathbb{R}^d)\Big)$. In this kind of tube, we compare X_ε to X_ε^φ solution of $dX_\varepsilon^\varphi = b(X_\varepsilon^\varphi, \xi_{t/\varepsilon}) \, dt + \sqrt{\varepsilon}\sigma(X_\varepsilon^\varphi) \, dW_t$, in other words, we will show that for all $\delta > 0$, for all continous function φ, there exist $\delta_1 > 0$ such that $P(\parallel X_\varepsilon - \varphi \parallel \leq \delta) \leq P(\parallel X_\varepsilon^\varphi - \varphi \parallel \leq \delta_1)$. It is easy to check it by using the exponential inequality. For absolutely continuous functions $\varphi \in \Big(\mathcal{C}([0,1], \mathbb{R}^d)\Big)$, the mapping $F^\varphi : \Big(\mathcal{C}([0,1], \mathbb{R}^d)\Big) \times B^0_{M_2,w_\alpha} \longrightarrow B^0_{M_2,w_\alpha}$ defined by

$$F^\varphi(g, f) = h \text{ if and only if } h_t = x + g_t + \sigma(\varphi_t)f_t + \int_0^t f_s d\sigma(\varphi_s)$$

is continuous and $X^{\varepsilon,\varphi}$ is the image of $(y^{\varepsilon,\varphi}, \sqrt{\varepsilon}W)$ by F^φ

$$\text{where } dy_t^{\varepsilon,\varphi} = b(y_t^{\varepsilon,\varphi}, \xi_{t/\varepsilon}) \, dt, \;\; y_0^{\varepsilon,\varphi} = 0. \tag{8}$$

Let $L^0(x, \alpha)$ be the conjugate of the quadratic convex function $H^0(x, \alpha)$ obtained from the formula in (2). L^0 is lower semicontinuous(lsc), with values in $\mathbb{R}_+ \cup \{\infty\}$, convex to second argument

For some couple values (φ, ψ) in $B([0,T], \mathbb{R}^d)$, denoted by :

$$\begin{cases} S^0(\varphi,\psi) & = \displaystyle\int_0^T L^0(\varphi_s, \dot{\psi}_s)\ ds \ \ if\ \psi\ is\ absolutly\ continuous \\ & = +\infty \ \ otherwise \end{cases}$$

$$\begin{cases} S^W(\psi) & = \displaystyle\int_0^T \frac{1}{2} \mid \dot{\psi}_s \mid^2 \ ds \ \ if\ \psi\ is\ absolutly\ continuous \\ & = +\infty \ \ otherwise \end{cases}$$

Proposition 4.2. *For absolutely continuous functions* $\varphi \in \Big(\mathcal{C}([0,1], \mathbb{R}^d)\Big)$ *then* $S^0(\varphi, .)$ *is a rate function of* $y^{\varepsilon,\varphi}(0)$ *in* $\Big(\mathcal{C}([0,1], \mathbb{R}^d)\Big)$*(see[10]).*

Proposition 4.3. *Assume (H4), the couple of random variables* $(y^{\varepsilon,\varphi}(0), \sqrt{\varepsilon}W)$ *considered a random variable with values in* $B^0_{M_2,w_\alpha}$ *satisfying LDP with the following rate function* $S^\varphi(g,f)$ *definied by :*

$$S^\varphi(g,f) = S^0(\varphi, g) + S^W(f) \tag{9}$$

By using the contraction principle, the law of X^φ_ε satisfies LDP on $B^0_{M_2,w_\alpha}$ with the rate function defined by :

$$S\varphi(\omega) = \inf\{S^\varphi(g,f), \omega = F^\varphi(g,f)\}. \tag{10}$$

Now we aim to establish in Theorem (4.1) a large deviation principle (LDP) for the family X_ε on the Besov-Orlicz Space $B^0_{M_2,w_\alpha}$ by using the Azenctott's method mentioned above to the random variables $X^\varepsilon_1 = \sqrt{\varepsilon}W$ and $X^\varepsilon_2 = X^\varepsilon$

Theorem 4.4. *For any* $r, \alpha, a > 0$, *for each* x *with values in* $\mathbb{R}^d$, *there exist* $\rho, \tilde{r}, \varepsilon_0$ *depending only on* r, α, a, x *such that for* g, f *absolutly continuous verifing* $\| f \| \leq a$ *and* $\varphi = B_y(g,f)$, $|x-y| \leq \tilde{r}$, $\varepsilon \leq \varepsilon_0$ *we have,*

$$\mathbb{P}\Bigg(\| X^\varepsilon(x) - \varphi \|_{M_2,w_\alpha} > \alpha, \| y^{\varepsilon,\varphi}(0) - g \| < \rho, \| \sqrt{\varepsilon}W - f \| < \rho\Bigg) \leq \exp(-\frac{r}{\varepsilon}).$$

where $\varphi = B_x(g,f)$ *if and only if* $\dot{\varphi}_t = \dot{g}_t + \sigma(\varphi)\dot{f}_t, \varphi_0 = 0$

Proof of Theorem 4.4. Indeed, let $W^f = W - \frac{1}{\sqrt{\varepsilon}}f$. Girsanov's theorem implies that W^f is a d-dimensional Wiener process with respect to the probability P^f given by

$$\frac{dP^f}{dP} = \exp\Big(\frac{1}{\sqrt{\varepsilon}}\int_0^1 \dot{f}_s\ dW_s - \frac{1}{\varepsilon}\int_0^1 |\dot{f}_s|^2\ ds\Big)$$

Let $\{Y_t^\varepsilon, 0 \leq t \leq 1\}$ be the solution of SDE

$$Y_t^\varepsilon = x + \int_0^t b(Y_s^\varepsilon, \zeta_{s/\varepsilon})\ ds + \sqrt{\varepsilon}\int_0^t \ \sigma(Y_s^\varepsilon)\ dW_t^f, P^f almost \ \ surely \tag{11}$$

To simplify the notation, set for any ρ ,α, $\varepsilon > 0$

$$U^f = \{\parallel X^\varepsilon(x) - \varphi \parallel_{M_2,w_\alpha} > \alpha, \parallel y^{\varepsilon,\Phi}(0) - g \parallel < \rho, \parallel \sqrt{\varepsilon}W - f \parallel < \rho\}$$

And

$$V^f = \exp\Big\{\Big|\frac{1}{\sqrt{\varepsilon}}\int_0^1 \dot{f}_s\ dW_s\Big| > \frac{\lambda}{\sqrt{\varepsilon}}\Big\}.$$

Then

$$\begin{aligned} P(U^f) \ &\leq P\Big\{U^f \cap \Big(V^f \leq \exp\Big(\tfrac{\lambda}{\varepsilon}\Big)\Big)\Big\} + P\Big\{V^f > \tfrac{\lambda}{\varepsilon}\Big\} \\ &\leq \exp\Big(\tfrac{\lambda + a/2}{\varepsilon}\Big)P^f(U^f) + P\Big(\Big|\tfrac{1}{\sqrt{\varepsilon}}\textstyle\int_0^1 \dot{f}_s\ dW_s\Big| \geq \tfrac{\lambda}{\varepsilon}\Big) \end{aligned} \tag{12}$$

where $a = \parallel h \parallel_H^2$ and $\lambda \in \mathbb{R}$
By the classical exponential inequality,

$$P\Big(\Big|\int_0^1 \dot{f}_s\ dW_s\Big| \geq \frac{\lambda}{\sqrt{\varepsilon}}\Big) \leq 2\exp(-\frac{\lambda^2}{2a\varepsilon}) \leq \exp(-\frac{r}{\varepsilon}). \tag{13}$$

Set

$$Y^\varepsilon(W^f) = X^\varepsilon(W^f + \frac{1}{\sqrt{\varepsilon}}f).$$

Consequently, we obtain :

$$P^f(U^f) = P\Bigg(\ \parallel Y^\varepsilon(x) - \varphi \parallel_{M_2,w_\alpha} > \alpha, \parallel y^{\varepsilon,\Phi}(0) - g \parallel < \rho, \parallel \sqrt{\varepsilon}W \parallel < \rho\Bigg),$$

where Y^ε is the solution of SDE in (11), the estimate (12) and (13) complete the proof of the theorem (4.4).

The aim of proof of theorem 4.4 is an immediate consequence of the next following propositions.
For any $n \in \mathbb{N}^*$ we consider the approximation sequence of the process Y^ε defined by

$$Y_t^{\varepsilon,n} = Y^\varepsilon_{\frac{j}{2^n}}, if \ \ s \in [\frac{j}{2^n}, \frac{j+1}{2^n}[\ \ for \ \ all \ j = 0, 1, 2, ..., 2^n - 1$$

Proposition 4.5. *For all* $r > 0$ *and* $\gamma > 0$ *there exist* $\varepsilon_0 > 0$ *and* n *such that if* $0 < \varepsilon < \varepsilon_0$*, we have :*

$$P^f\Big\{\parallel Y^\varepsilon - Y^{\varepsilon,n} \parallel \geq \gamma\Big\} \leq \exp(-\frac{r}{\varepsilon})$$

Proof of Proposition 4.5. For a detailed proof of Proposition 4.5, we refer to Priouret,P(1982, Lemma 2) [21]

Proposition 4.6. *For every* $\gamma_1 > 0$, $\rho > 0$ *then*

$$P^f(U^f) \leq P^f(\| \sqrt{\varepsilon} \int_0^{\cdot} \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon \|_{**} > \gamma_1, \| \sqrt{\varepsilon} W^\varepsilon \| < \rho).$$

Proof of Proposition (4.6).

$$\begin{aligned} Y_t^\varepsilon - \varphi &= x - y + \int_0^t [b(Y_s^\varepsilon, \xi_{s/\varepsilon}) + \sigma(Y_s^\varepsilon)\dot{f}_s]\ ds + \sqrt{\varepsilon}\int_0^t \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon \\ &\quad - \int_0^t [b(\varphi_s, \xi_{s/\varepsilon}) + \sigma(\varphi_s)\dot{f}_s]\ ds + y_t^{\varepsilon,\varphi} - g_t \\ &= x - y + \int_0^t [b(Y_s^\varepsilon, \xi_{s/\varepsilon}) - b(\varphi_s, \xi_{s/\varepsilon})]\ ds + \int_0^t [\sigma(Y_s^\varepsilon) + \sigma(\varphi_s)]\dot{f}_s\ ds \\ &\quad + \sqrt{\varepsilon}\int_0^t \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon + y_t^{\varepsilon,\varphi} - g_t \end{aligned}$$

Denote by $I_t^\varepsilon = \sqrt{\varepsilon}\int_0^t \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon$, let $\delta > 0$ be such that $\|I_t^\varepsilon\| \leqslant \delta, \|x - y\| \leqslant \tilde{r}$

$$\begin{aligned} \|Y_t^\varepsilon - \varphi\| &\leqslant \tilde{r} + C\int_0^t |Y_s^\varepsilon - \varphi_s|\ ds + C\int_0^t |Y_s^\varepsilon - \varphi_s|\ |\dot{f}_s|\ ds + \|I_t^\varepsilon\| + \|y_t^{\varepsilon,\varphi} - g_t\| \\ &\leqslant \tilde{r} + C\int_0^t |Y_s^\varepsilon - \varphi_s|(1 + |\dot{f}_s|)\ ds + \|I_t^\varepsilon\| + \|y_t^{\varepsilon,\varphi} - g_t\| \end{aligned}$$

An application of Gronwall's lemma implies that,

$$|Y_t^\varepsilon - \varphi_t| \leq (\tilde{r} + \| y^{\varepsilon,\varphi} - g \| + \| \sqrt{\varepsilon}\int_0^{\cdot} \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon \|) \exp\Big(C(\int_0^t (1 + |\dot{f}_s|\ ds))\Big).$$

On the one hand

$$\begin{aligned} \| Y_t^\varepsilon - \varphi_t \|_{**} &\leq \| \sqrt{\varepsilon}\int_0^{\cdot} \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon \|_{**} + \| y^{\varepsilon,\varphi} - g \| \\ &\quad + \| \int_0^t [b(Y_s^\varepsilon, \xi_{s/\varepsilon}) + \sigma(Y_s^\varepsilon) f_s]\ ds - \int_0^t [b(\varphi_s, \xi_{s/\varepsilon}) + \sigma(\varphi_s) f_s]\ ds \|_{**} \\ &\leq \|\sqrt{\varepsilon}\int_0^{\cdot} \sigma(Y_s^\varepsilon)\ dW_s^\varepsilon\|_{**} + \|y^{\varepsilon,\varphi} - g\| + \| \int_0^t [b(Y_s^\varepsilon, \xi_{s/\varepsilon}) - b(\varphi_s, \zeta_{s/\varepsilon})]\ ds\|_{**} \\ &\quad + \| \int_0^t [\sigma(Y_s^\varepsilon) - \sigma(\varphi_s)]\dot{f}_s\ ds\|_{**} \\ &\leq \|I_t^\varepsilon\|_{**} + \|y^{\varepsilon,\varphi} - g\| + C\int_0^t \|Y_s^\varepsilon - \varphi_s\|_{**}(1 + |\dot{f}_s|)\ ds \\ &\leq \|I_t^\varepsilon\|_{**} + \|y^{\varepsilon,\varphi} - g\| + \sup_{0\leq u\leq v\leq 1} \frac{C}{w(u - v)}\int_u^v (1 + |\dot{f}_s|)|Y_s^\varepsilon - \varphi_s|\ ds \end{aligned}$$

On the other hand, knowing the fact that

$$|Y^\varepsilon_s-\varphi_s|\le |Y^\varepsilon_u-\varphi_u|+|(Y^\varepsilon_s-\varphi_s)-(Y^\varepsilon_u-\varphi_u)|$$

$$\begin{aligned}\|Y^\varepsilon-\varphi\| &\le \|I^\varepsilon_s\|+\|y^{\varepsilon,\varphi}-\varphi\|+C(1+|f|)\|Y^\varepsilon-\varphi\|+C\int_0^t(1+|\dot{f}_s|)\|Y^\varepsilon-\varphi\|_{**}\,ds\\ &\le 2\delta+C(1+|f|)\|Y^\varepsilon-\varphi\|+C\int_0^t(1+|\dot{f}_s|)\,ds\end{aligned}$$

Now, by using Grownall's lemma, we obtain

$$\|Y^\varepsilon-\varphi\|\le 2\delta\Big[1+C(1+|f|)e^{C(1+|f|)}\Big]e^{C(1+|f|)}$$

Thus :

$$P^f(U^f)\le P^f(\|\sqrt{\varepsilon}\int_0^\cdot \sigma(Y^\varepsilon_s)\;dW^\varepsilon_s\;\|_{**}>\gamma_1,\|\sqrt{\varepsilon}W^\varepsilon\;\|<\rho).$$

Proposition 4.7. *For all $r>0,\gamma_1>0$, there exist $\varepsilon>0$ and $\rho>0$ such that*

$$P^f(\|\sqrt{\varepsilon}\int_0^\cdot \sigma(Y^\varepsilon_s)\;dW^\varepsilon_s\;\|_{**}>\gamma_1,\|\sqrt{\varepsilon}W^\varepsilon\;\|<\rho)\le \exp(-\frac{r}{\varepsilon}).$$

Proof of Proposition (4.7). For $\alpha>0$ and for every $n\in\mathbb{N}$, we have

$$A=\Big\{\;\|\sqrt{\varepsilon}\int_0^\cdot \sigma(Y^\varepsilon_s)\;dW^\varepsilon_s\;\|_{**}\ge\rho,\|\sqrt{\varepsilon}W\;\|\le\alpha\Big\}\subset A_1\cup A_2\cup A_3$$

where

$$\begin{cases}A_1=\Big\{\;\|\sqrt{\varepsilon}\int_0^\cdot[\sigma(Y^\varepsilon_s)-\sigma(Y^{\varepsilon,n}_s)]dW^\varepsilon_s\;\|_{**}\ge\frac{\rho}{2},\|\;Y^\varepsilon-Y^{\varepsilon,n}\;\|\le\gamma\Big\}\\ A_2=\Big\{\;\|\;Y^\varepsilon-Y^{\varepsilon,n}\;\|\ge\gamma\Big\}\\ A_3=\Big\{\;\|\sqrt{\varepsilon}\int_0^\cdot\sigma(Y^{\varepsilon,n}_s)\;dW_s\;\|_{M_2,w}\ge\frac{\rho}{2},\|\sqrt{\varepsilon}W\;\|\le\alpha\Big\}\end{cases}$$

By using the Proposition 4.5, we obtain : for all $r>0$ and $\gamma>0$ there exist ε_0 and n such that for every $0<\varepsilon<\varepsilon_0$, we have :

$$P^f(A_2)\le\exp(-\frac{r}{\varepsilon}\;)$$

It is easy to check that if $\|\;Y^\varepsilon-Y^{\varepsilon,n}\;\|\le\gamma$ we get $\|\;\sqrt{\varepsilon}[\sigma(Y^\varepsilon_s)-\sigma(Y^{\varepsilon,n}_s)]\;\|_{**}\le 4\varepsilon M^2\gamma^2$.
By using the lemma (3.8),

$$P^f(A_1)\le C\exp\Big(-\frac{\rho^2}{C\gamma^2\varepsilon}\Big)$$

It should increase $P^f(A_3)$. So we have

$$\begin{aligned}\| \sqrt{\varepsilon}\int_0^{\cdot} \sigma(Y_s^{\varepsilon,n})\ dW_s \|_{M_2,w} &= \sqrt{\varepsilon} \| \sum_{j=0}^{n} \sigma(Y_{t_j}^{\varepsilon,n})[W(t_{j+1}\wedge .)-W(t_j\wedge .)] \|_{M_2,w} \\ &\leq \sqrt{\varepsilon}\sum_{j=0}^{n} \| \sigma(Y_{t_j}^{\varepsilon,n})[W(t_{j+1}\wedge .)-W(t_j\wedge .)] \|_{M_2,w} \\ &\leq 2\sqrt{\varepsilon}Kn \| W \|_{**}\ .\end{aligned}$$

By using the lemma (3.7), we have :

$$P^f(A_3) \ \leq \ C\max\Big(1,\big(\frac{\rho}{16lMn\alpha}\big)^2\Big)\exp\big(-\frac{\rho^2}{C\varepsilon 16M^2n^2}\log\big(\frac{\rho}{16lMn\alpha}\big)\Big)$$

where C is a constant depending on l et M.
Let $r>0$ et $\rho>0$, we choose then $\gamma>0$ small enough that $\frac{\rho}{C\gamma^2}>r$, and n such that

$$P^f(A_1)\leq C\exp\big(-\frac{r}{\varepsilon}\big)$$

and finally $\Big(\frac{\rho^2}{16M^2n^2}\log\big(\frac{\rho}{16lMn\alpha}\big)\Big) > Cr$ in (14). This ends the proof of the proposition.

4.1 Construction of the rate function

For any $(x,\alpha)\in(\mathbb{R}^d)^2$, denote by $H(x,\alpha)=H^0(x,\alpha)+\frac{1}{2}\langle\alpha,\Sigma_x\alpha\rangle$ the quadratic function associated to $\sigma(x)$ so $\Sigma_x=\sigma(x)^t\sigma(x)$.
Let us suppose that $L(x,\beta)$ the conjugate quadratic function of $H(x,\alpha)$. L is lower semicontinuous function with values $\mathbb{R}_+\cup\{+\infty\}$, converged to β, verified by the following : for all $\varphi,\psi\in B([0,T],\mathbb{R}^d)$, we denote by

$$S(\varphi,\psi)=\begin{cases}\displaystyle\int_0^T L(\varphi_s,\psi_s)\ ds, \text{ if } \psi \text{ is an absolutly continuous,}\\ +\infty \text{ otherwise.}\end{cases} \tag{14}$$

Theorem 4.8. *For the absolutely continuous R^d-value function ψ, let be $S(\varphi,\psi)$ the formula defined in* (14) *and $S^\varphi(\psi)$ the rate function defined in* (10)*. Then there exist a couple of absolutly continuous functions (g,f) verified by $\psi=F^\varphi(g,f)$ and we obtain $S(\varphi,\psi)$ and $S^\varphi(\psi)$ coincide.*

Proof of theorem 4.8. We denote for $(x,\alpha)\in(\mathbb{R}^d)^2$, $H(x,\alpha)=H^0(x,\alpha)+\frac{1}{2}\langle\alpha,\Sigma_x\alpha\rangle$. Q_x denotes the quadratic form on $\mathbb{R}^n$ associated with the matrix $\sigma(x)$,

defined by $Q_x(v) = \langle v, \sigma(x)\sigma(x)^* v\rangle = \inf |w|^2$, $\sigma(x)w = v$, $v \in \mathbb{R}^n$.
We denote for $(x, \beta) \in (\mathbb{R}^d)^2$,

$$L(x, \beta) = \inf\{L^0(x, \gamma) + Q^*(\delta); b(\gamma) + \delta = \beta\}$$

where Q^* is the quadratic form Q_x.
Let $\tau = B_x(g, f)$ be the solution of $\dot{\tau}_t = b(\dot{g}_t) + \sigma(\tau_t)\dot{f}_t$

$$\begin{aligned} S^0(\varphi, g) + S^W(f) &= \int_0^T L^0(\varphi_s, \dot{g}_s) + \frac{1}{2}|\dot{f}_s|^2 \, ds \\ &\geqslant \int_0^T L^0(\varphi_s, \dot{g}_s) + \frac{1}{2}\inf\{|\dot{g}_s|^2; \sigma(\tau_s)\dot{g}_s\nabla_s\} \, ds \\ &\geqslant \int_0^T L^0(\varphi_s, \dot{g}_s) + \frac{1}{2}Q^*_{\varphi_s}(\nabla_s) \, ds \\ &\geqslant \int_0^T \inf\{L^0(\varphi_s, \dot{g}_s) + \frac{1}{2}Q^*_{\varphi_s}(\nabla_s); b(\dot{g}_s) + \nabla_s = \dot{\tau}_s\} \, ds \\ &\geqslant \int_0^T L^0(\varphi_s, \dot{\tau}_s) \, ds \end{aligned}$$

So,

$$S^{\varphi}(\tau_s) \geqslant S(\varphi, \tau).$$

To check the other inequality, consider $A_x[v]$ defined by

$$A_x[v] = \{w \text{ tel que } \sigma(x)w = v, \ v \in \mathbb{R}^n\}$$

Consider the Borel set Γ defined by

$$\Gamma = \{(x, v) \in U \times \mathbb{R}^n \text{ such that } A_x[v] \text{ is not empty}\}$$

For each $(x, v) \in \Gamma$, we put

$$K(x, v) = \{w \in \mathbb{R}^n \text{ such that } |w| = \inf |u|; u \in A_x[v]\}$$

The mapping $K : \Gamma \to \{\text{compact in } \mathbb{R}^k\}$ is a measurable family of non-empty compact toward so Rockafeller [25]. Subsequently, there exist a Borelian function $\chi : \Gamma \to \mathbb{R}^k$ such that $\chi(x, v) \in K(x, v)$ for $(x, v) \in \Gamma$.
For each φ, ψ such that $S(\varphi, \psi) < +\infty$ we put Ω as set of (x, β) such that $L(x, \beta) < +\infty$ such as

$$S(\varphi, \psi) = \int_0^T L(\varphi_s, \psi_s) \, ds$$

As

$$Q_x(v) = \langle v, \sigma(x)\sigma(x)^* v\rangle = \|\sigma^*(x)v\|^2$$

and

$$Q^*_x(v) = \inf\{|w|^2, w \in A_x[v]\},$$

we have

$$Q^*_{\varphi_s}(\varphi'_s - b(\varphi_s)) = |\chi(\varphi_s, \varphi'_s - b(\varphi_s))|^2$$

$$S(\varphi, \psi) = \int_0^T L(\varphi_s, \psi_s)\ ds = \int_0^T \inf\{L^0(\varphi_s, \dot{g}_s) + \frac{1}{2}Q^*_{\varphi_s}(\nabla_s); b(\dot{g}_s) + \nabla_s = \dot{\tau}_s\}\ ds.$$

So there exist a functional $f \in C^0(\mathbb{R}^k)$ such that

$$S(\varphi, \psi) \leqslant \int_0^T \inf\{L^0(\varphi_s, \dot{g}_s) + \frac{1}{2}|\dot{f}|^2\}\ ds.$$

It is fair enough to ask $\dot{f}_s = |\chi(\varphi_s, \nabla_s)|$ for almost everything $s \in [0, T]$.

4.2 Regularity of the solution in the Besov-Orlicz space

It is clear that the process $\displaystyle\int_0^t b(X^\varepsilon_s, \zeta_{s/\varepsilon})\ ds, t \in I$ belongs a.s. to $B^{\varphi,0}_{M_2,w}$. Then, it remains to show that the process $\displaystyle\int_0^t \sigma(X^\varepsilon_s)\ dW_s, t \in I$ satisfies (1.1) and (1.2). We will prove the result in the case $k = d = 1$. The extension in the general case is easily deduced

Let us put

$$Y_t = \int_0^t \sigma(X^\varepsilon_s)\ dW_s$$

We will show that for some p_0, we have for any $\alpha < \dfrac{1}{2}$

$$\sup_{j \geqslant 0} \sup_{p \geqslant p_0} \frac{2^{-j/p}}{p^{1/2}(1+j)^\alpha} \Big[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \Big]^{1/p} < \infty \quad p.s. \tag{15}$$

$$\lim_{j \vee p \to p_0} \frac{2^{-j/p}}{p^{1/2}(1+j)^{1/2}} \Big[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \Big]^{1/p} = 0 \tag{16}$$

To check the relation 15, let $\lambda > 0$. Using Chebychev inequality, we can get

$$P\Big(\frac{2^{-j/p}}{p^{1/2}(1+j)^\alpha} \Big[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \Big]^{1/p} > \alpha \Big) \leqslant \frac{\lambda^{-p}\ 2^{-j}}{\sqrt{\frac{p}{2}}(1+j)^{\alpha p}} \Big(\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p \Big)$$

$|A_n(Y)|$ is dominated by the terms of :

$$A := \Big| \int_0^t f_{\frac{2k-1}{2^{j+1}}, \frac{2k}{2^{j+1}}}(s)\ dW_s \Big| \quad \text{et} \quad B := \Big| \int_0^t f_{\frac{2k-2}{2^{j+1}}, \frac{2k-1}{2^{j+1}}}(s)\ dW_s \Big|,$$

where

$$f_{r,t}(s) = 1_{r<s\leqslant t}\sigma(t, X_s) + 1_{s<r\leqslant t}[\sigma(t, X_s) - \sigma(r, X_s)].$$

For integers $p \geqslant 2$, using the inequality of Barlow-Yor(1982), for A and B, there exist a constant C_p appearing in the Burkholder-Davis-Gundy inequality such that

$$E|A_n(Y)|^p \leqslant CM^p p^{p/2}.$$

Hence,

$$\begin{aligned} P\Big(\frac{2^{-j/p}}{p^{1/2}(1+j)^{\alpha}}\Big[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p\Big]^{1/p} > \alpha\Big) &\leqslant \frac{\lambda^{-p}\ 2^{-j}}{\sqrt{\frac{p}{2}}(1+j)^{\alpha p}}\Big(\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p\Big) \\ &\leqslant \Big(\frac{C}{\lambda}\Big)^p \frac{1}{(1+j)^{\alpha p}} \end{aligned}$$

Choosing $p_0 \geqslant \dfrac{1}{\alpha}$ and λ large enough, the series

$$\sum_{j\geqslant 0}\sum_{p\geqslant p_0}\Big(\frac{C}{\lambda}\Big)^p \frac{1}{(1+j)^{\alpha p}}$$

converges. The point (15) is then a consequence to Borel-Cantelli's lemma.
To prove 16, we have to notice that as above $|A_n(Y)|$ is dominated by terms of the form A et B the exponential inequalities yield that there exist positive constants K_1 et K_2 such that for all $\lambda > 0$ large enough,

$$P\Big(\frac{1}{\sqrt{1+j}}\sup_n |A_n(Y)| > \alpha\Big) \leqslant K_1 \exp\frac{-\lambda^2(1+j)}{K_2 M^2}\ .$$

Therefore, the Borel-Cantelli's lemma leads to

$$\sup_{j\geqslant 1}\frac{1}{\sqrt{1+j}}\sup_n |A_n(Y)| < \infty \quad p.s.$$

Or

$$2^{-j/p}\Big[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p\Big]^{1/p} \leqslant \sup_n |A_n(Y)|$$

Thus

$$\sup_{j\geqslant 1}\frac{2^{-j/p}}{p^{1/2}(1+j)^{1/2}}\Big[\sum_{n=2^j+1}^{2^{j+1}} |A_n(Y)|^p\Big]^{1/p} \leqslant \frac{1}{p^{1/2}}\sup_{j\geqslant 1}\sup_n |A_n(Y)|.$$

and that ends the establishment of (16).

4.3 Conluding remarks

In the present paper, we have etablished a large deviation principle (LDP) associated of stochastic differential equation solution of (1) in the Besov-Orlicz space by using Azenctott's method. This extends the LDP proved by H.LAPEYRE [17] to the case of usual topology of uniform convergence. A natural extension of this work is to replace the standard brownian motion by a Fractional brownian motion W^H for every value of the Hurst parameter $H \in (0,1)$

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