Primaries Oblateness effects on the Collinear Libration Points in the restricted-three body problem

Abstract

In this work, the canonical Hamiltonian form of the restricted three- body problem including the effects of primaries oblateness is presented. Moreover, the collinear libration points are obtained. In addition to this, the relation between position of libration points and variation in (mass ration , oblateness coefficients A1 and A2) is studied. The results obtained are a good agreement with Perdios [5] & Singh [8]. The Poincare surface section PSS is used to illustrate the stability of motion around each of the collinear libration points. A numerical application on the real system Earth-Moon is presented.

Key Words: restricted three body problem, oblateness effects, Poincare surface of section, stability motion, libration points.

Introduction

A special case of general three body problem is the restricted three body problem (RTBP) which play an important role in celestial mechanics. The equations of motion, in general case are non–linear in nature and it is more difficult to obtain analytic solutions for the general problem. Hence some restrictions were put to overcome this problem as the circular and elliptical RTBP. The restricted three-body problem with oblate primaries has also attracted the interest of many Researches [1-4]. Perdios [5] investigated the combined influence of the oblateness and radiation pressures of the primaries on collinear points moreover calculated Lyapunov planar and 3D family of periodic orbits around these points. Markello [4] obtained zero velocity and libration points for Hill's problem with the effect of oblateness. Abdul Raheem [6] found the periodic orbits around the triangular libration points under the effects of the centrifugal and the Coriolis forces together with solar radiation and oblateness of the two primaries. Singh [7] studied the nonlinear stability of the triangular points when both primaries are oblate spheroids. Ibrahim [9] obtained the Lissajous orbits and the phase spaces around collinear points under the effect of oblateness. The stability of a test particle moves about the equilibrium points in the circular restricted three-body problem

is studied with both primaries oblateness and radiation together with P–R drag[10]. Analysing LSS in the Earth-Moon system, exploring dynamical structures that are available within a multi-body gravitational environment are investigated [11]. The Laplace transformations are used to study the effect of mass variation on the stability of libration points of the restricted three-body problem [12]. Equations of motion for the perturbed circular restricted three-body problem are regularized in canonical variables in a moving coordinate system, two different L-matrices of the fourth order are used in the regularization [13].

The perturbed mean motion n of the primaries is given by $n^2 = 1 + \frac{3}{2}(A1 + A2)$, where $Ai = \frac{r_{ei}^2 - r_{pi}^2}{5\,R^2}$ is the oblateness coefficient of m_1 and m_2 having the equatorial and polar radii as r_{ei} and r_{pi}, respectively and R is separation between the primaries. Then the effective potential equation with oblateness of two primaries is given by

$$V(x, y, z) = \frac{n^2}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)A1}{2r_1^3} + \frac{\mu\,A2}{2r_2^3} \tag{1}$$

Where

$$r_1 = \sqrt{(x - \mu)^2 + y^2} \tag{2.1}$$

$$r_2 = \sqrt{(x + 1 - \mu)^2 + y^2} \tag{2.2}$$

The Hamiltonian system of R3BP with Oblateness

The Hamiltonian of the restricted three body problem with oblateness can be written as

$$\mathcal{H} = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) + yp_x - x\,p_y - \frac{n^2}{2}(x^2 + y^2) - \left[\frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)A1}{2r_1^3} + \frac{\mu\,A2}{2r_2^3}\right] \tag{3}$$

It is clear that Equation [3] has more two terms than the previous Hamiltonian obtained by many authors, which are $\frac{(1-\mu)A1}{2r_1^3}$ and $\frac{\mu\,A2}{2r_2^3}$ and the mean motion n depends on the oblateness.

58 The canonical form is given by

59 $$\dot{x} = \frac{\partial \mathcal{H}}{\partial p_x} = p_x + y \qquad (4.1)$$

60 $$\dot{y} = \frac{\partial \mathcal{H}}{\partial p_y} = p_y - x \qquad (4.2)$$

61 $$\dot{z} = \frac{\partial \mathcal{H}}{\partial p_z} = p_z \qquad (4.3)$$

62 $$\dot{p}_x = -\frac{\partial \mathcal{H}}{\partial x} = -p_y - n^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu\ (x-\mu+1)}{r_2^3} - \frac{3A1\ (1-\mu)\ (x-\mu)}{2r_1^5} - \frac{3\ A2\ \mu\ (x-\mu+1)}{2\ r_2^5} \qquad (4.4)$$

63 $$\dot{p}_y = -\frac{\partial \mathcal{H}}{\partial y} = p_x - n^2 y - \frac{(1-\mu).\ y}{r_1^3} - \frac{\mu\ y}{r_2^3} - \frac{3\ A1\ (1-\mu)y}{2\ r_1^5} - \frac{3\ A2\ \mu\ y}{2\ r_2^5} \qquad (4.5)$$

64 $$\dot{p}_z = -\frac{\partial \mathcal{H}}{\partial z} = -\frac{(1-\mu).\ z}{r_1^3} - \frac{\mu\ z}{r_2^3} - \frac{3\ A1\ (1-\mu).\ z}{2\ r_1^5} - \frac{3\ A2\ \mu\ z}{r_2^3} \qquad (4.6)$$

65 Equations (4) represent the equation of motion of the third body under the effect of

66 gravitational forces and the Oblateness of two primaries.

Location of the Libration points with effect of Oblateness

68 To obtain the location of libration points, put

69 $p_x = p_y = p_z = \dot{p}_x = \dot{p}_y = \dot{p}_z = 0$, then eqns. (4.4), (4.5) and (4.6) will be

70 $$-n^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu\ (x-\mu+1)}{r_2^3} - \frac{3A1\ (1-\mu)(x-\mu)}{2r_1^5} - \frac{3\ A2\ \mu\ (x-\mu+1)}{2\ r_2^5} = 0 \qquad (5.1)$$

71 $$-n^2 y - \frac{(1-\mu).\ y}{r_1^3} - \frac{\mu\ y}{r_2^3} - \frac{3\ A1\ (1-\mu)y}{2\ r_1^5} - \frac{3\ A2\ \mu\ y}{2\ r_2^5} = 0 \qquad (5.2)$$

72 $$-\frac{(1-\mu).\ z}{r_1^3} - \frac{\mu\ z}{r_2^3} - \frac{3\ A1\ (1-\mu).\ z}{2\ r_1^5} - \frac{3\ A2\ \mu\ z}{r_2^3} = 0 \qquad (5.3)$$

73 The collinear points can be determined from equation (5.1).

74 $$n^2 x = \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu\ (x+\mu-1)}{r_2^3} - \frac{3A1\ (1-\mu)(x+\mu)}{2r_1^5} - \frac{3\ A2\ \mu\ (x+\mu-1)}{2\ r_2^5} \qquad (6)$$

75 The locations of the collinear points are

76 $$x_1 = \mu - 1 - X1, \qquad x_2 = \mu - 1 + X2, \qquad x_3 = \mu + X3.$$

77 whereX1, X2 and X3 satisfy ninth degree polynomials while in Ibrahim [14] it was

78 seventh degree when one of primary has oblate spheroid. By expanding equation

79 (6) up to sixth power in x, then the first point is obtained by solving the next

80 equation which gives only one real root, this real root is the location of libration

81 point

82 $$X1^9(2 + 3A1 + 3A2) + X1^8(10 + 15A1 + 15A2 - 2\mu - 3A1\mu - 3A2\mu) +$$

83 $$X1^7(20 + 30A1 + 30A2 - 8\mu - 12A1\mu - 12A2\mu) + X1^6(18 + 30A1 + 30A2 -$$

84 $$8\mu - 18A1\mu - 18A2\mu) + X1^5(6 + 15A1 + 15A2 + 4\mu - 12A1\mu - 12A2\mu) +$$

85 $$X1^4(6A1 + 3A2 + 12\mu - 6A1\mu) + X1^3(8\mu + 12A2\mu) + X1^2(2\mu + 18A2\mu) +$$

86 $$X1(12A2\mu) + 3A2\mu = 0 \qquad\qquad (7)$$

87 Similarly for the second and third points

88 $$(2 + 3A1 + 3A2)X2^9 + (-10 - 15A1 - 15A2 + 2\mu + 3A1\mu + 3A2\mu)X2^8 +$$

89 $$(20 + 30A1 + 30A2 - 8\mu - 12A1\mu - 12A2\mu)X2^7 + (-18 - 30A1 - 30A2 +$$

90 $$8\mu + 18A1\mu + 18A2\mu)X2^6 + (6 + 15A1 + 15A2 + 4\mu - 12A1\mu -$$

91 $$12A2\mu)X2^5 + (-6A1 - 3A2 - 12\mu + 6A1\mu)X2^4 + (8\mu + 12A2\mu)X2^3 +$$

92 $$(-2\mu - 18A2\mu)X2^2 + 12A2x\mu X2 - 3A2\mu = 0 \qquad\qquad (8)$$

93

94    $(2 + 3A1 + 3A2)X3^9 + (8 + 12A1 + 12A2 + 2\mu + 3A1\mu + 3A2\mu)X3^8 +$

95    $(12 + 18A1 + 18A2 + 8\mu + 12A1\mu + 12A2\mu)X3^7 + (10 + 12A1 + 12A2 +$

96    $8\mu + 18A1\mu + 18A2\mu)X3^6 + (10 + 3A1 + 3A2 - 4\mu + 12A1\mu + 12A2\mu)X3^5 +$

97    $(12 - 3A1 - 12\mu + 6A1\mu)X3^4 + (8 - 12A1 - 8\mu + 12A1\mu)X3^3 +$

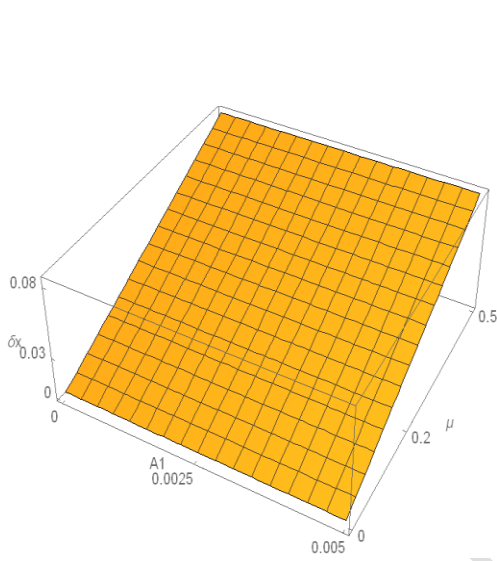98 $(2 - 18A1 - 2\mu + 18A1\mu)X3^2 + (-12A1 + 12A1\mu)X3 - 3A1 + 3A1\mu = 0$ (9)

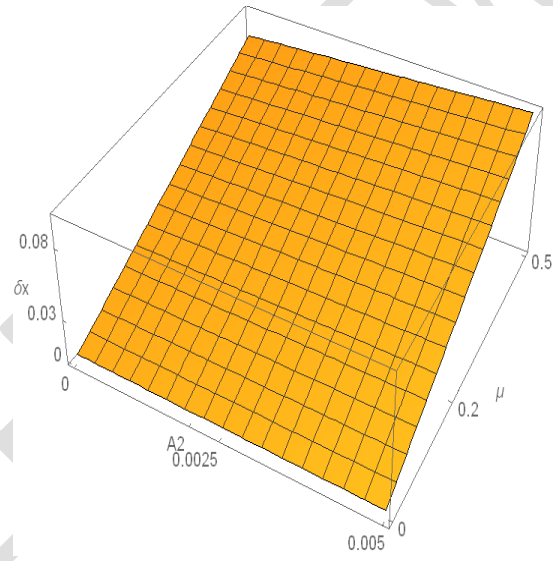99

100   **fig 1.a variation of L1 under the effect of A1**   **fig 1.b variation of L1 under the effect of A2**

101
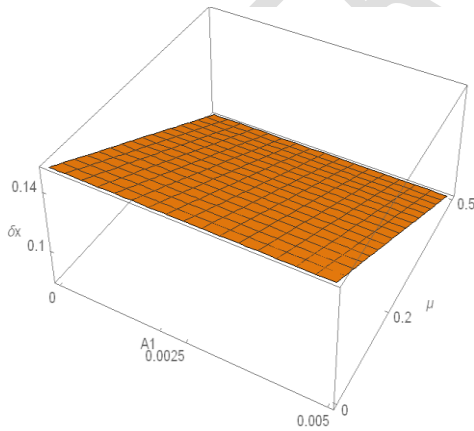
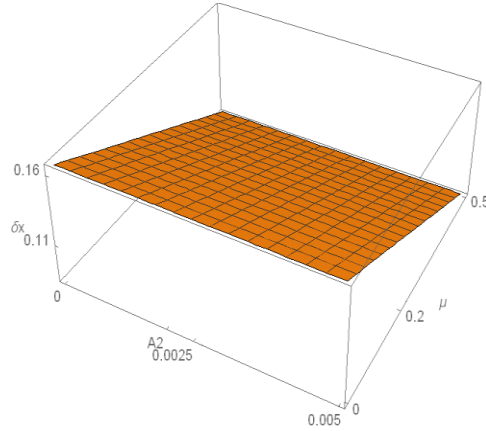102   **fig 2.a variation of L2 under the effect of A1**   **fig 2.b variation of L2 under the effect of A2**
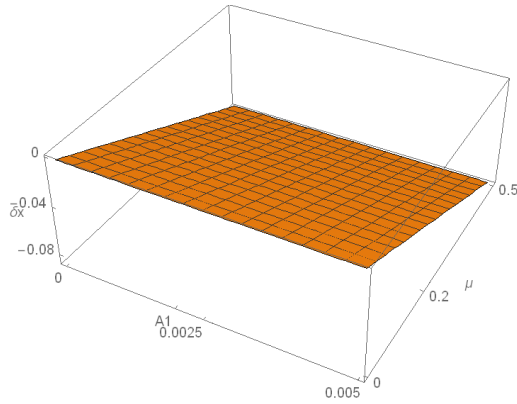
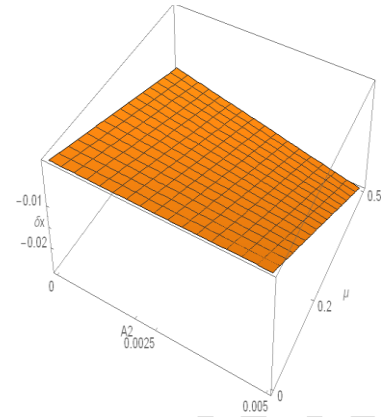fig 3.a variation of L3 under the effect of A1   fig 3.b variation of L3 under the effect of A2

Figures 1–3 show surface representations of the variation of collinear libration (L1, L2, L3) under the effects of oblateness for the bigger and smaller primaries where $\mu \in (0, .5]$, A1 and A2 $\in [0, 0.005]$. Where fig 1 and fig 3 represent the increase in variation of L1 and L3 when μ, A1 and A2 increase contrary with fig2 the decrease in variation of L2 when μ, A1 and A2 increase. this coincide with [1], [2].

Poincare surfaces of section

In the restricted three-body problem, Poincare surface of Section (PSS) is a powerful technique for studying stability of orbits which enables finding stable periodic and quasiperiodic around the two primaries. The four-dimensional phase space $(x, y, \dot{x}, \dot{y})$ is used to obtain (PSS) of the infinitesimal body at any instant. This is a good tool to study the stability of nature system which enables determine the regular or chaotic nature of the trajectory. Figures (4) and (5) show Poincare surface section, when $A1 = A2 = 0$ and $A1 \neq 0, A2 \neq 0$ for Earth-Moon

119   system. The numerous islands can be observed which means that the behavior of

120   the trajectory is likely to be regular, where the curves shrink down to a point, it

121   represents a periodic orbit. To obtain this PSS an initial condition of x varies from

122   0.6 to 0.8 with $\delta x = 0.001$ are used. So that, the canonical equations of motions (4)

123   are integrated truncated up to 1000 steps using Runge–Kutta fixed step sized
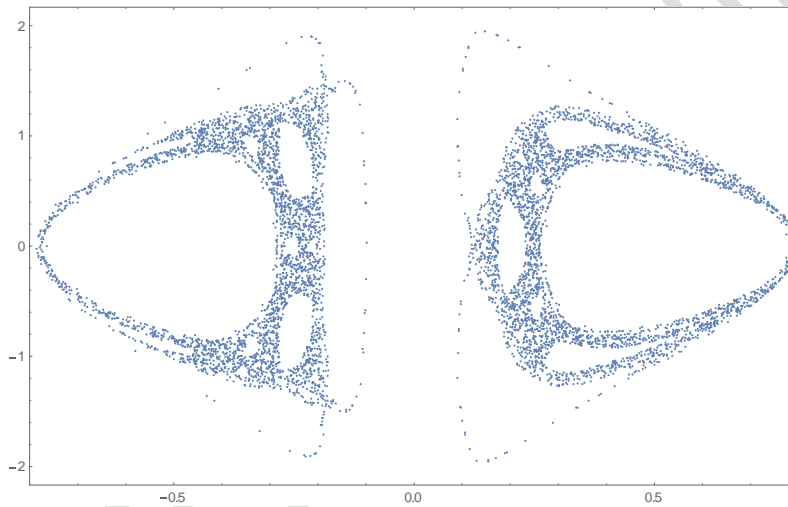
124   method.

125

126   fig. 4 Poincare surface sections for earth-moon system A1= A2 = 0
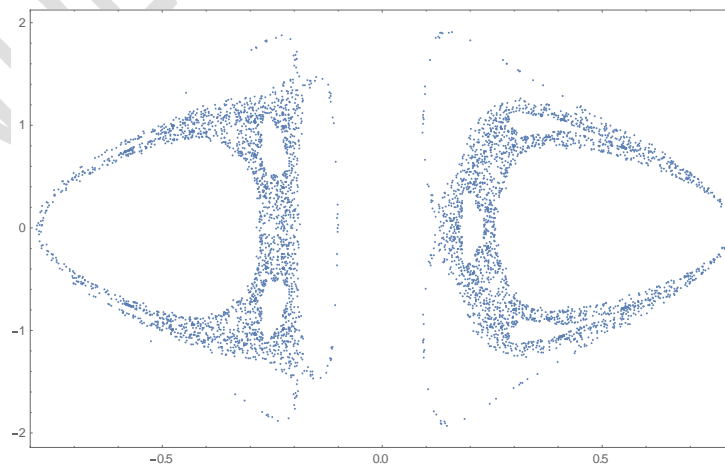
127

128

129   Fig. 5 Poincare surface sections for earth-moon system A1≠A2 ≠0
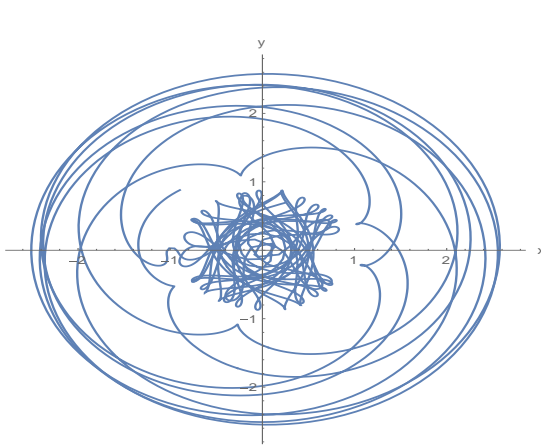
130

131
132

Figure 6.a Time from 0 to 200
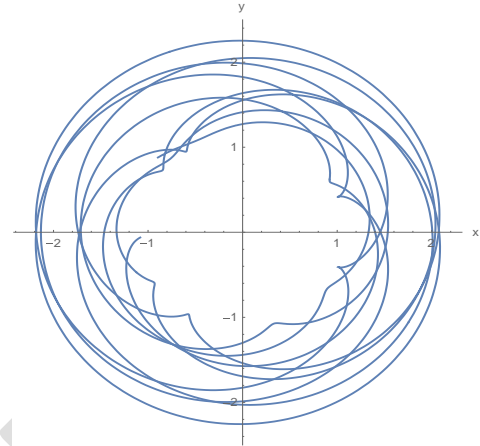

figure 6.b Time from 200 to 324

133
134
135
136
137

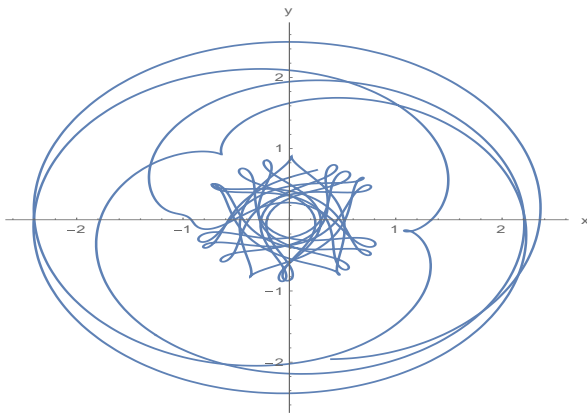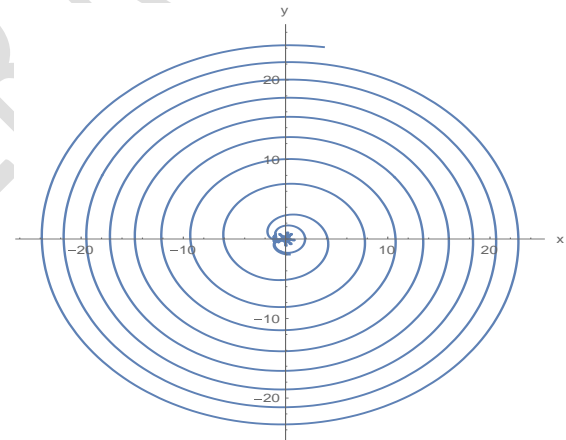figure 7.a t from 0 to 100

figure 7.b t from 100 to 200

138

139
140
141

Figs.6, and Figs.7 represent the orbits of the third body. Figs.6 displays the orbit in presence of oblateness effect while Figs.7 displays orbit in absence of oblateness effect. The orbit of the infinitesimal body represents in first frame when $0 \leq t \leq 200$ whereas second frame when $200 \leq t \leq 340$. In the second frame, in the case of absence of oblateness effect, the orbit as spiral shape. However, with effect of oblateness, orbit becomes regular when $0 \leq t \leq 200$ which is shown in third frame while fourth frame shows the orbit when $100 \leq t \leq 200$.

149

150

151

Conclusion

This study related to the motion of the third body under the effects of the oblateness of the two primaries. It is found that the positions of L2 decreased under the effects of oblateness while the positions of L1 and L3 are increased under this effect. A PSS shows the stability of nature system which are a regular motion in the intervals where x belongs to].15, 0.8[and]-0.2,-.8[, while $\dot{x}$ belongs to]-1.8,1.8[, and a chaotic motion are investigated in the intervals x belongs to]-.15, 0.15[while $\dot{x}$ belongs to]-1.8,-2[and]1.8,2[for the system without oblateness and under the effects of oblateness. These kinds of study are very important for the space missions. In near future the study will truncated under the effects of solar radiation pressure and the oblateness which is very important for the motion of solar sails.

Refernces

[1] Szebehely, V.G. Theory of Orbits: The Restricted Problem of Three-Bodies. (1967), Academic Press Inc., New York.

[2] Sharma, R.K. and SubbaRao, P.V. Celes. Mech. (1976),13, 137.

[3] SubbaRao, P.V. and Sharma, R.K.: Celes.Mech. Dyn. Astron. (1997),65, 291.

[4] Markellos, V. V, Roy, A. E., Perdios, E. A. & Douskos, C. N. A Hill Problem with Oblate Primaries and Effect of Oblateness on Hill Stability of Orbits. (2000). 295–304

[5] Perdios, G. Computation of the Liapunov Orbits in the Photogravitational RTBP with Oblateness. Astrophys Space Sci (2006) 305:389–398.

[6] Abdulraheem, A. & Singh, J. Combined effects of perturbations, radiation and oblateness on the periodic orbits in the restricted three-body problem Combined effects of perturbations, radiation and oblateness on the periodic orbits in the restricted three-body problem. Astrophys Space Sci (2008) 317: 9–13.

[7] Singh, R. Combined effects of perturbations, radiation, and oblateness on the nonlinear stability of triangular points in the restricted three-body problem, Astrophys Space Sci (2011)

[8] Singh, R. & Amuda, T. O. Binaries application around the collinear equilibrium points in the photogravitational CR3BP with bigger primary oblate. Eur. Phys. J. Plus (2016) 131: 137

[9] Ibrahim, A.H., Ismail, M.N., Zaghrout, A.S., Younis, S.H. and El Shikh, M.O. Lissajous Orbits at the Collinear Libration Points in the Restricted Three-Body Problem with Oblateness. World Journal of Mechanics, (2018). 8, 242-252

184   [10] Singh, J., and T. O. Amuda. "Perturbation effects in the generalized circular restricted

185 three-body problem." Indian Journal of Physics 92.11 (2018): 1347-1355.

186 [11] Bucci, Lorenzo, et al. "Periodic orbit-attitude solutions along planar orbits in a perturbed

187 circular restricted three-body problem for the Earth-Moon system." *Acta Astronautica* 147

188 (2018): 152-162.

189 [12] Younis, Sahar H., Fatma M. Elmalky, and M. N. Ismail. "Variation Mass Effects on the

190 Stability of Libration Points of Restricted Three Body Problem (Laplace

191 Transformation)." *Journal of Advances in Mathematics and Computer Science* (2018): 1-13.

192 [13] Poleshchikov, S. M. "Regularization of the Perturbed Spatial Restricted Three-Body Problem by L-

193 Transformations." *Cosmic Research* 56.2 (2018): 151-163.

194 [14] Ibrahim. A. H., Ismail.M.N., Zaghrout. A. S., Younis. S. H. and El Shikh. M. O.

195 Orbital Motion Around the Collinear Libration Points of the Restricted Three-Body Problem,

196 Journal of Advances in Mathematics and Computer Science, (2018) 29(1),1-16

197