

Using of the application programs for the decision of tasks on physics

Abstract

The article discusses the possibilities of using modern computer technologies, in particular, the Mathcad application program for visual representation of physical processes. This article shows methods for solving ordinary differential equations in the Mathcad package based on numerical methods. As an example of a nonlinear process, the Cauchy problem for a second order ordinary differential equation is solved using the Mathcad odesolve function, which is a complication of the linear oscillator equation, and a graph is obtained.

It also talks about the advantages of using the Mathcad application program for solving problems in physics, which allows you to not only make the necessary calculations, but also to arrange your work using graphs, drawings, tables and mathematical formulas. Based on the results of these modeling works, the user gets the system model ready and can only set the initial conditions and control all the parameters of the model during the numerical experiment. In this regard, this program provides an opportunity to expand the teaching activities of the teacher and increase the independence and activity of students.

Keywords: physical processes; MathCad; modeling; physical models; Runge-Kutta method; complex systems; solutions of ordinary differential equations; learning process; examples of problem solving.

1. Introduction

In modern conditions of intensive development of information technology, there is a need to create a new system (methods, forms, etc.) of the educational environment. Currently, the urgent issue is the use of program-pedagogical and telecommunication facilities in the educational process of a higher educational institution, and, in particular, in teaching physics.

Since the introduction of information technology in the educational process, the role of their use has increased. Physics as an educational discipline lends itself perfectly to the process of computerization. Information technology in the process of teaching physics can be used to study theoretical material, training, as a means of modeling and visualization, as well as in solving physical problems.

One of the main disciplines in technical universities is "Physics". Solving problems in physics classes contributes to the formation and consolidation of acquired knowledge and skills in practice in order to use them in professional activities. It is applied physical problems that reflect the technical content and essence of the future professional activities of a university graduate. The solution to this type of problem allows students to get acquainted with the various principles of the operation of technical devices, physical research methods [Kondratiev, 2015].

However, in the educational process, students encounter many difficulties and the use of computer technology in solving problems greatly facilitates and solves these difficulties. And also computer technologies in the process of solving problems develop students' interest in the subject of physics and computer technology.

Computer programs help students solve applied physical problems. Many tasks contain huge calculations, in the solution of which, by making a small mistake, you can get

the wrong answer. Computer programs make it possible to prevent such errors and make it possible to come to the correct answer quickly enough. The use of such programs can reduce the time for calculations, and increase the time spent on analysis and conclusion.

To date, computer programs that help solve applied physical problems are many. Many of them allow the calculation of tasks; some of them make charts, graphs and diagrams for which students would spend a lot of time; others allow virtual experiments and experiments, that is, many physical phenomena are clearly demonstrated to students.

Modern computer programs and telecommunication technologies provide students with access to non-traditional sources of information - electronic hypertext textbooks, application software packages, distance learning systems, etc., this is designed to increase the efficiency of development of cognitive independence and provide new opportunities for creative personal growth.

The development of information and telecommunication technologies is so fast that the existing pedagogical research does not have time to analyze new methods, means and forms of teaching physics [Levitskij, 2016].

The use of computers in the study of physics provides great opportunities. For example, computers are increasingly being used in laboratory work. More and more, the so-called virtual laboratory work is being introduced into the educational process in universities. Many programs such as Matcad, Matlab and others are used to process the results, which leads to a reduction in the time for this calculation of results and more time is left for better laboratory work. The use of computers in teaching physics allows you to change the teaching methodology, and this leads to the facilitation of the work of the teacher. A personal computer is not replaced by traditional teaching aids, but supplementing them and together with them form a system of teaching aids focused on the use of new information technologies, the use of which creates the conditions for teaching physics in the educational information environment.

Computer labs satisfy almost all the requirements of a physical experiment, except that they are not familiar with specific devices. Further, they can be used in distance learning. Computer labs can be used when a real physical experiment is not possible at all. For example, in the implementation of thought experiments that play an important role in the development of physics. They can also be used when the material is very complex and to study it requires increased visibility. It is computer labs that have such increased visibility. During computer experiments, the studied physical processes are visualized, and due to the application of computer modeling methods, graphic symbols on the screen depicting physical objects move in accordance with the laws of physics. However, these computer experiments were created precisely as interactive laboratory work, i.e. students in the course of their implementation can independently change the values of physical parameters and take measurements of "virtual" physical quantities, and then by calculating or plotting to determine other physical quantities. These "virtual" laboratory work can be performed along with the usual ones, they are not alternative, but mutually complement each other. In most of these laboratory works, phenomena are studied, the study of which is difficult using ordinary field works. [Bursian, 2015]

First of all, it is extremely convenient to use computer models in a demo version when explaining new material or in solving problems. Problem solving is a necessary element of teaching physics and the formation of a creative personality. Using a computer in the process of solving physical problems allows not only to better absorb physics, but also demonstrates the importance of a computer as an instrument of creativity and as an effective assistant in the study of the laws of nature.

2. Numerical methods for solving ordinary differential equations (odes)

As you know, any physical phenomena are described by differential equations, so the solution of ordinary differential equations for modeling physical processes is of great practical importance. The solution of ordinary differential equations based on numerical methods is widely used in the practice of scientific and technical calculations, as well as in solving various problems of physics, mechanics and other natural Sciences.

The main form of setting the initial conditions, which is used for modeling real processes using ordinary differential equations, is associated with determining the values of all lower derivatives at the starting point of the variable change interval. So, in physics, ordinary differential equations usually describe the change in the studied characteristic over time, and the initial conditions are determined at the moment $t = 0$. [Computers, models, calculable experiment (2016)]

Thus, given ordinary differential equations and systems of differential equations are called Cauchy tasks, that is, if additional conditions are given for a single value of an independent variable, then such a task is called a Cauchy task.

Methods for numerical solution of ordinary differential equations in the form of the Cauchy task are developed in great detail. The most popular of them are deservedly the Runge-Kutta algorithm, which is successfully used for solving the vast majority of differential equations.

To solve an ordinary differential equation, you need to know the values of the dependent variable and the derivatives for some values of the independent variable. If the conditions are set for two or more values of an independent variable, the problem is called a boundary value problem. Such tasks require finding a function (or several functions) of a single variable, if, first, a differential equation (or system of equations) containing the derivative of the function is defined, and, second, the necessary number of additional conditions specifying the value of the function at some starting point [Guld, K., & Tobochnik, 2017]

Solving Cauchy tasks for ordinary differential equations is a technology that has been developed in detail for a long time. With "good" ordinary differential equations, no computational problems usually arise at all (most often they are solved using the Euler and Runge—Kutta algorithm), and for a special type of ordinary differential equations, called hard ones, special methods must be used. All these features are embedded in Mathcad, and the user is allowed to choose a specific solution algorithm.

The Cauchy task can be formulated as follows: let be an ordinary differential equation:

$$\frac{dy}{dx} = f(x, y) \quad (1)$$

$y(x)$ initial condition $y(x_0) = y_0$. You need to find a function $y(x)$ that satisfies both the specified equation and the initial condition.

The numerical solution of the Cauchy task consists in constructing a table of approximate values of y_1, y_2, \dots, y_n for solving the equation $y(x)$ at points x_1, x_2, \dots, x_n . Most often $x_i = x_0 + ih, i = 0, 1, \dots, n$, where h is the increment step of the variable x , n is the number of intervals of the solution with a step h .

Here we consider two groups of numerical methods for solving the Cauchy task: one-step and multi-step.

One-step methods are methods where finding the next point on the $y(x) = f(x)$ curve requires information about only one previous step. The simplest one step method is the Euler method:

$$y_{i+1} = y_i + f(x_i, y_i)h \quad (2)$$

$i = 0, 1, \dots, n-1$.

The Euler method has low accuracy (on the order of h).

To achieve higher accuracy (order h^4), the fourth-order Runge-Kutta method is used:

$$y_{i+1} = y_i + \frac{k_0 + 2k_1 + 2k_2 + k_3}{6}, \text{ где, } \begin{aligned} k_0 &= h \cdot f(x_i, y_i), & k_2 &= h \cdot f(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}), \\ k_1 &= h \cdot f(x_i + \frac{h}{2}, y_i + \frac{k_0}{2}), & k_3 &= h \cdot f(x_i + h, y_i + k_2) \end{aligned}$$

Based on the above, now let's try to solve physical problems using Matcad based on numerical methods.

3. The result of numerical modeling

The solution of linear differential equations and their systems in Mathcad is presented in two forms: as a computational block and as inline functions. The first form is preferable from the point of view of visual representation of the solution and technical simplicity, while the second opens up much wider possibilities [Glushakov, Zhakin, Khachirov 2017]

The differential equation of the first order can by definition contain, in addition to the function $y(t)$ itself, only its first derivative $y'(t)$. In the vast majority of cases, the differential equation can be written in standard form (Cauchy form):

$$y'(t) = f(y(t), t) \quad (3)$$

and only with this form can the Mathcad computing processor work. Correct from a mathematical point of view, the formulation of the corresponding Cauchy task for first-order ordinary differential equations must contain one initial condition in addition to the equation itself - function value $y(t_0)$ the value of the function x at some point t_0 . You need to explicitly define the function $y(t)$ on the interval from t_0 to t_x . By the nature of the performance the Cauchy tasks are also called tasks with initial conditions [Diyakonov, 2015]

To numerically integrate a single ordinary differential equation, the Mathcad user has the choice of either using the given/odesolve computing block, or using built-in functions, such as the rkfixed function, as in previous versions of Mathcad. The first way is preferable for reasons of visual representation of the problem and results, and the second gives the user more leverage over the parameters of the numerical method.

Computing block Given/Odesolve

A computational block for solving a single ordinary differential equation that implements the Runge-Kutta numerical method consists of three parts:

- Given-keyword;
- the ordinary differential equation and the initial condition written using logical operators must be typed on the Boolean toolbar (Boolean operators) and the initial condition must be in the form of $y(t_0) = b$;
- odesolve $x(t, t_1)$ - inline function for solving an ordinary differential equation with respect to a variable t on an interval (t_0, t_1) , with $t_0 < t_1$

It is acceptable, and even often preferable, to set the Odesolve function $(t, t_1, step)$ with three parameters, where $step$ is an internal parameter of the numerical method that

determines the number of steps in which the Runge - Kutta method will calculate the solution of the differential equation. The larger the step, the more accurate the result will be, but the more time will be spent searching for it. Keep in mind that selecting this parameter can significantly (several times) speed up calculations without significantly impairing their accuracy [Turin, V. O., Markov, O. I., Poyarkov, 2018]

An example of solving the Cauchy task for a first order ordinary differential equation $y' = y - y^2$ using a computational block is given in listing 1

Listing 1. Solution of the Cauchy problem for an ordinary differential equation of the first order

Given

$$y(0) = 0.1$$

$$y'(t) = y(t) - y(t)^2$$

$$y := \text{Odesolve}(t, 10)$$

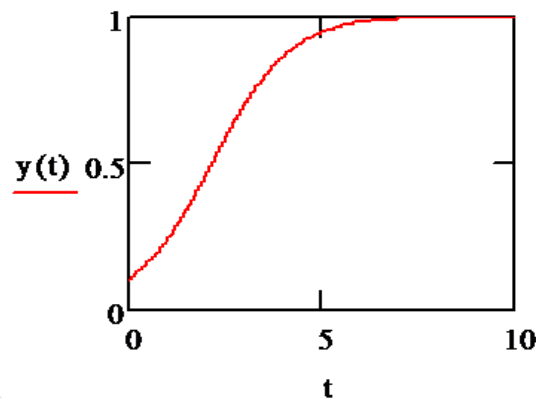


Fig. 1. Solution of the Cauchy task

Keep in mind that you should insert Boolean operators using the Boolean operators toolbar. When entering from the keyboard, remember that the logical equal sign corresponds to the keyboard shortcut <Ctrl>+<=>. The derivative symbol can be entered using the Calculus panel. Mathcad requires that the end point of integration of an ordinary differential equation lies to the right of the initial one: $t_0 < t_1$ (in listing 1. $t_0 = 0, t_1 = 10$), otherwise an error message will be returned.

As you can see, the result of using the Given/odesolve block is a function $y(t)$ defined on the interval (t_0, t_1) . You should use the usual Mathcad tools to plot it or get the function value at some point in the specified interval, for example: $y(3) = 0.691$. The user has the option to choose between two modifications and the Runge-Kutta numerical method. To change the method, right-click on the odesolve function area to open the context menu and

select one of two options: Fixed (Fixed step) or Adaptive (Adaptive). By default, the first one is used, i.e. the Runge-Kutta method with a fixed step. [Verzhbitsky2001]

Now let's try using the method of solving equations using the `odesolve` function to simulate various processes, in particular the model of a nonlinear harmonic oscillator. It is based on the solution of the Cauchy problem for an ordinary differential equation of the second order, which is a complication of the equation $(2\omega y'' + \beta y' + y + \gamma y^2 = 0)$ of a linear oscillator, where ω is the cyclic frequency of oscillations, γ is the attenuation coefficient. The harmonic oscillator model describes, in particular, the pendulum oscillations: $y(t)$ describes changes in the angle of its deviation from the vertical, $y'(t)$ - the angular speed of the pendulum, $y''(t)$ - acceleration, and the initial conditions, respectively, the initial deviation of the pendulum $y(0) = 1.0$ and the initial speed $y'(0) = 0$. It is important to note that the model is linear, that is, the unknown function (and its derivatives) are included in the equation in the first degree [Gorbatenko, 2019]

The method for solving the equation for the nonlinear case is given in Listing 2, and the result is shown in Fig.2. Here the symbol of the derivative is allowed to enter the tools panel, Calculus (Computing). Once again, we emphasize that the result of applying the Given/odesolve block is the function $y(t)$ defined on the interval (t_0, t_1) . You should use the usual Mathcad tools to plot it or get the function value at some point in the specified interval, for example: $y(10) = 0.048$.

Listing 2. Modeling of a nonlinear oscillator

$$w := 0.5 \quad \beta := 0.2 \quad \gamma := 0.95$$

Given

$$w^2 \cdot \frac{d^2}{dt^2} y(t) + \beta \cdot \left(\frac{d}{dt} y(t) \right) + y(t) + \gamma \cdot y(t)^2 = 0$$

$$y(0) = 0 \quad y'(0) = 3$$

$$y := \text{Odesolve}(t, 20)$$

$$y(10) = 0.048$$

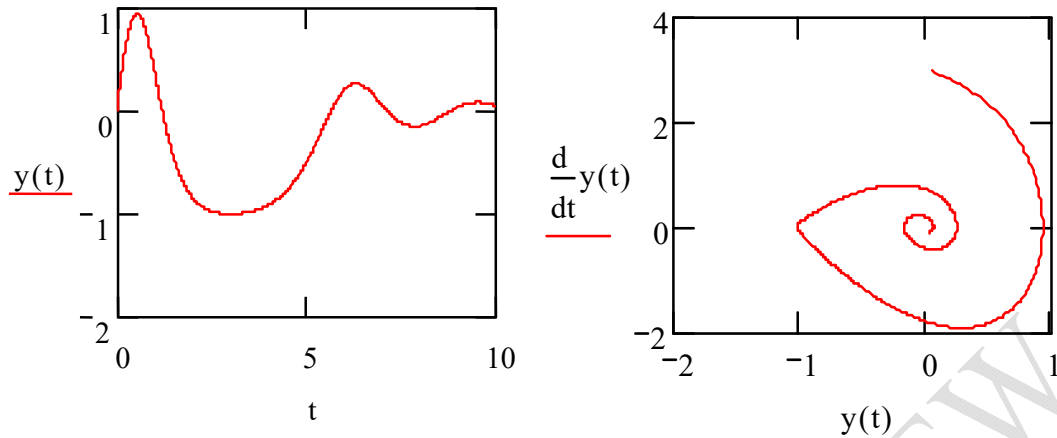


Fig 2. The model of nonlinear harmonic oscillator

As Listing 2 shows, in addition to the equation itself, it was necessary to define two initial conditions (the third and fourth lines of the listing)— initial values of $y(t)$ and $y'(t)$ for $t=0$. Generally speaking, ordinary differential equations have a unique solution if, in addition to the equation, the initial or boundary conditions are specified in a certain way

The resulting nonlinear model is widely used in the study of sections of physics: mechanics, molecular, electricity and magnetism, vibrations and waves. Nonlinear systems with dynamic chaos are used in systems of hidden information transmission, as well as in communication systems that use dynamic chaos as a source of vibrations that carry information.

From the above, it follows that the use of modern application packages in the educational process allows you to significantly change the methodology of studying some issues of the physics course related to conducting cumbersome, repetitive computational procedures, solving systems of differential equations, plotting graphs and surfaces, with a visual representation of the results of solving the problem using application packages. If before the behavior of a physical system was analyzed exclusively analytically, now it is possible to use numerical methods of computer simulation, which have certain advantages.

Computer modeling, conducting a computational experiment is one of the modern methods of studying physical phenomena. It has its own characteristics, advantages and disadvantages compared to other methods of studying physical systems. It is quite obvious that students of higher educational institutions should have ideas about computer models, numerical methods for studying various objects of cognition, and be fairly free to navigate in modern software products. It is modern application packages that make it possible to solve a complex system of equations in a few seconds, construct a graph of the studied dependence, and simulate a difficult reproducible experiment.

The advantages of modern packages are expressed in providing the ability to enter mathematical formulas or functions for numerical calculation by them, setting various values of the quantities used, plotting graphs for a visual representation of the simulation results, generating random variables (modeling random processes), performing logical operations, which allows you to implement various numerical methods. Using Mathcad, the student does not waste time coding the computational algorithm and programming auxiliary blocks i.e. saves the student from the mass of routine computing work. Also, its advantages are that Mathcad makes studying physics easier, the Mathcad program itself is easy to learn, and does not require reading thick books, conducting abstracts and memorizing complex rules for studying and applying. Mathcad is simple in that a solution to a problem of interest can be

obtained in a short period of time. In this regard, the Mathcad package is very effective in the educational process, makes it possible to teach a number of educational disciplines (computer modeling of physical processes, physics, mathematical modeling, numerical methods, etc.) at a higher level.

The results of the work can be used when reading courses of computer modeling of physical processes, when conducting practical, lecture, laboratory classes. The complex complements traditional forms of teaching (lectures, seminars, physical laboratory) and can be used in computer classes of all universities, technical, pedagogical institutes and other higher educational institutions as a modern addition to physics courses.

Conclusion

Thus, in this paper, as an example of a nonlinear process, the Cauchy problem for a second-order ordinary differential equation was solved using the `odesolve` function of the Mathcad package, which is a complication of the linear oscillator equation, and a graph was obtained. Based on the results of these model works, the user gets a ready-made model of the system and can only set the initial conditions at random and control all the parameters of the model during the numerical experiment. In this regard, this program provides an opportunity to expand the teaching activities of the teacher and increase the independence and activity of students.

Solving the problem of a General physics course using Mathcad allows you to form ideas about the possibilities of using this mathematical package for further research. Using the Mathcad program not only reduces the time for laboratory work, but also activates the process of studying the discipline by students, facilitates and accelerates the assimilation of new material and control, which ultimately improves the quality and deepens students' knowledge.

The integration of physics, mathematics, and computer technologies and the creation of a whole set of exercises and tasks, and tasks that are specific, rather than abstract from practice, will allow you to achieve a deeper understanding of the physical foundations and a more purposeful and meaningful development of the mathematical apparatus.

This application of mathematical modeling can lead to huge cost savings and a significant reduction in research time. Mathematical modeling for monitoring and evaluating design solutions and experimental methods created not only significantly improves the quality of design solutions, but also dramatically reduces the cost of creating experimental installations and conducting scientific research using them.

References

1. Kondratiev, A.S. (2015). Fizika. Zadachy na kompiutere [Physics. Tasks on a computer]. Publishing house MSTU [in Russian].
2. Levitskij, A.A. (2016). Matlab 3.05, MathCad 2.5. Krasnayarsk: publishing house Phismatlit [in Russian].
3. Bursian, E. V. (2015). Zadachy dlyakompiutera [Tasks on physics for a computer] M.: Inlightening [in Russian].
4. Guld, K., & Tobochnik, Y. (2017). Kompiuternoemodirovanie v fizike [A computer design is in physics]. M.: World [in Russian].
5. Chekalin A.N., Konyukhov V.M., Konyukhov I. V. Computer Simulation and Comparison of the Efficiency of Conventional, Polymer and Hydrogel Waterflooding of Inhomogeneous Oil Reservoirs // Eurasian Chemico-Technological Journal. — 2017. — Vol. 19, Is. 4. — P. 323–334.
6. Glushakov, S.V., & Zhakin, I.A., & Khachirov, T.S. (2017) Matematicheskoe modelirovanie Mathcad 2000, Matlab 5.3 [Mathematical design Mathcad 2000, Matlab 5.3]. Khar'kov; Folio, M.: OOO « Publishing house AST ».[in Russian].
7. Diyakonov, V. P. Spravochnik po MathCAD PLUS 7.0 PRO [Reference book on MathCAD PLUS 7.0 PRO] M.: SK Press [in Russian].
8. Turin, V. O., Markov, O. I., Poyarkov, V. N. Aberration of light and matter wave. // Modern problems of physical and mathematical Sciences: Materials of the IV all-Russian scientific and practical conference with international participation. November 22-25, 2018, Orel: I. S. Turgenev OSU, - Pp. 199-207.[in Russian].
9. Samarsky A. A., Mikhailov A. P. Mathematical modeling: Ideas. Methods. Examples. GL. ed. Fiz. Mat. lit., - Moscow: Nauka 2001. - 320 p.[in Russian].
10. Methodological developments using mathematical packages. Electron. Dan. Mode of access: <http://www.exponenta.ru>. [in Russian].
11. Verzhbitsky V. M. Numerical methods. Mathematical analysis and ordinary differential equations, Moscow: Higher school of Economics, 2001, 383 p. 12. E. A. Volkov. Numerical method. Saint Petersburg: LAN, 2004. - 248 p.[in Russian].
12. Gorbatenko, A. I. Organization of the educational process based on the application of innovative teaching methods in an agricultural University // Physics and modern technologies in agriculture. Materials of the X International youth conference of young scientists, students and schoolchildren. Orel: EBS Orelgau, 2019. - Pp. 353-358. 612[in Russian].
13. Tarasova, M. A. Interdisciplinary integration of academic disciplines – an effective technology for the formation of professional and active components of competencies / Tarasova, M. A., Grishina, S. Yu. // Scientific notes of the Oryol state University. Scientific journal. Series "Natural, technical and medical Sciences". - 2014-No. 5 (61), OSU publishing house, Pp. 409-412.[in Russian].