Original Research Article

Modeling of Seasonal Multivariate Time Series analysis; Using Gross Domestic Product (GDP) in Nigeria (January 1985- 2017)

ABSTRACT

The multivariate "Seasonal Vector Autoregressive Moving Average" was used to measure the growth rate of Gross Domestic Product (GDP) in five (5) sectors: Agriculture, Industries, Building/Construction, Wholesales/Retails, and Services. The data was gathered from the National Bureau of Statistics and spans 33 years, from 1985 to 2017. To evaluate the model, real (R) software was used. The variability statistics for the five variables show that all of the variables have a seasonality pattern that is not stationary. We difference the data series (once) to obtain stationary series and define the season order to indicate the seasonality pattern. We find the best model using Akaike information criteria and Bayesian information criteria. The best model was determined to be the SVARMA (4, 1, 1) $(1, 0, 0)_{12}$. We also apply model simplification to the SVARMA (4, 1, 1) $(1, 0, 0)_{,\ 12}$ model, to exclude statistically insignificant parameters. The forecasts revealed that the rate of growth in the Agriculture sector is slowly growing, the rate of growth in the Industries sector is slowly decreasing, the rate of growth in the Building/Construction sector is increasing, the rate of growth in the Wholesales/Retails sector is not stable, and the rate of growth in the services sector is poor.

Keyword: Agriculture, Industries, Building/Construction, Wholesale/Retail, Services, GDP, and Seasonal.

INTRODUCTION

For several years, economics researchers have been involved in forecasting (predicting) data activity. For this reason, a variety of forecasting methods have been proposed and developed. The number of researchers to investigate will be quantified using data analysis techniques and analytical technologies. Nigeria's economy has not been stable over the years, and as a result, the country has been plagued by economic challenges, threats, and shocks, both internal and external, for decades. Internally; the product of spending and consumption patterns, as well as a lack of effective public policy substitution and perception causes changes. Externally, crises may be caused by population growth, revolution, or war, among other things. Every country's economic growth reflects its ability to increase service and goods production. It refers to an increase in a country's Gross Domestic Product (GDP). Macroeconomic variables play a key role in any country's economic output. Nigeria's economy has faced a range of challenges in both the agricultural and non-agricultural sectors, all of which have led to a slowing of growth, which may have an impact on GDP. As a result, this study aims to look into the interrelationships of Nigeria's GDP across these sectors. Some factors, such as agriculture, industry, wholesale retail, building & services, will be taken into account. This research work on the seasonal multivariate time series model for the sectors in Nigeria's GDP will help to improve macroeconomic policy formulation in Nigeria especially by predicting the future trend of output from major sectors like Agriculture, Manufacturing industries, General services. The Central Bank of Nigeria (CBN), National Bureau of Statistics (NBS), Ministry of Finance, and ministry of planning will hopefully gain from the application of findings of this work in their research and statistics departments periodically. The results of this study will reawaken interest in the development of quantitative skills for statistical economic and financial analysis

Reviews of related work

Multivariate time series models are very crucial in modeling and identifying the joint structure on which these decisions depend. The resulting seasonal multivariate analyses provide good insight into the multivariate structure and also a simple guide to model choice and assessment.

Practically, an elaboration of this basic model is to incorporate time-variation in covariance matrices [9], recently in economics, an application was devoted to forecasts US employment growth [10]

Multivariate analysis is suitably applied in making such a forecast. These techniques have benefited from big improvements with regard to the easiness of use [4]. [6] Conducted research on stock index forecasting: a comparison of classification and level estimation on multivariate models, assessing the effectiveness of several multivariate techniques to group the level of estimation method, and comparing the relative measurement intensity of the models with respect to the trading benefit produced by their forecast. They also came to the conclusion that applying the model's threshold trading rules increases returns.

[5] Examined the seasonality of hip fracture and estimates of season-attribution effects using a multivariate ARIMA analysis of population-based evidence. The findings of their study, which used the autoregressive integrated moving average (ARIMA) model, revealed that seasonality and month have a major impact on hip fracture admission rates. According to the ARIMA regression coefficients, hip fractures are often more frequent in January and May.

[11] Researched the seasonal ecology of recent benthic Ostracoda from the North Cadiz Gulf coast using multivariate analysis (Southwestern Spain). They look at the seasonal components of ostracodes, and find that, as compared to previous studies, some recovery has degraded the system. [7] conducted a multivariate approach to modeling univariate time series using an autoregressive model, the model allowed for periodically varying coefficients and adopts vector elements in integrated the maximum likelihood method in cointegration check with the annual series. The researcher also concluded that it is often to apply transformation for the non-stationary seasonal time series in order to obtain better results. [3] Looked at the relationship between inflation, work rates, and GDP using multivariate time series analysis. The results of the multivariate time series analysis using STATA software revealed that the inflation rate has no effect on GDP, while the work rate has a negative relationship with GDP. The causality between

the variables in the study was also determined using Granger causality. According to their findings, all independent variables have a unidirectional relationship with GDP in the short term. [6] Investigated how oil price shocks affect investment using a multivariate vector autoregressive model with impulse response function and other experiments. Oil price shocks have a negligible effect on real GDP, according to the findings. They also came to the conclusion that oil price shocks have no absolute effect on actual GDP. To make such a prediction, the multivariate model is used. These methods have benefited from significant advancements in terms of ease of use. [16] Looked at checking for non-linearity in multivariate time series. The researcher considered a multivariate extension of the test proposed by [14] to ignore non-linearity, which used main components to resolve the test's dimensionality problem. An adaption of multivariate analysis to new technologies such as databases, the internet, economic data, etc. is an emerging area.

Methodology

SEASONAL OCCURRENCE

Mostly, seasonality showed in many economic, financial, and environmental variables. However, this can also occur in many earnings per share of the organization, which exhibit the characteristics of the yearly cyclic method. The unemployment number of a country always show the effect as many searches for a job mostly at the end of a year as many students graduated from school. Similarly, we can also observe this pattern on the daily temperature of a given location in Nigeria, we can also note this in the rate of traveling and the rate of consumption of Natural gas is also seasonal. So we can also see that many economic data published in the Central Bank of Nigeria bulletin are seasonally adjusted. The X-12 model procedure is based on most adjustment techniques, except this model has a seasonal frequency trough, which implies that some seasonality remains in the results. For example, the monthly unemployment rate of the United State is analyzed in, [15] (chapter three). So, in implementations, even for seasonally adjusted results, it is important to consider seasonal models.

MULTIVARIATE TIME SERIES

Suppose a K-dimensional vector of time series $w_t = (w_{1t}, w_{2t}, w_{3t} \ldots w_{kt})$.

A multivariate model is the (kx1) vector (w_t) where the i^{th} row of (w_t) is (w_{it}). This implies that for any time t, $w_t = (w_{1t}, w_{2t}, w_{3t} \ldots w_{kt})$.

Linearity of multivariate Time Series W_t

Statistically, speaking multivariate model is nonlinear; moreover linear series can often give an accurate approximation for making a decision.

$$W_t = \mu + \sum_{i>0}^{\infty} \psi a_{t-i} \quad (3.1)$$

Inevitability of multivariate model

A multivariate series $\{w_t\}$ is a linear encounter of its lagged values, hence, multivariate time series is always a value of the model w_t as a tool of its lagged values w_{t-i} for i is greater than 0 plus new information at time t. This can be presented as

$$w_t = c + a_t + \sum_{j=1}^{\infty} \pi_j w_{t-j} \ , \tag{3.2}$$

This equation must be a convergent series and the invertibility condition of the model is that all π must be less than 1 in a unit circle.

STATIONARY PROCESS

A probability process is said to be stationary if its 1st and 2nd moments are dependent on time. That is, a stochastic process w_t is stationary, if

i. $E(w_t) = \mu \ \ for\ all$ (3.3)

ii. $E\ [(w_t - \mu)(w_{t-k} - \mu)'] \ = cov(w_t\ \ )$ (3.4)

This implies that $\Gamma_z(k) \Rightarrow \Gamma_z(-k)'$ for t and $k\ = 0,1,2,$ ….

VECTOR AUTOREGRESSIVE (VAR) MODEL

In modeling dynamics between a set of variables, the VAR model provides us with an approach. This method is specifically concerned with the dynamics of multiple variables. This can be written as;

$$\mathrm{w_t} = \theta_0 + \ \theta_1 \mathrm{w_{t\text{-}1}} + \theta_2 \mathrm{w_{t\text{-}2}} + \cdots + \theta_p \mathrm{w_{t\text{-}p}} + \mathrm{a_t} \tag{3.5}$$

The VAR (P) model can be written in the matrix form as

$$\begin{pmatrix} w_{1t} \\ \\ w_{2t} \\ \vdots \\ w_{kt} \end{pmatrix} = \begin{pmatrix} \theta_{10} \\ \\ \theta_{20} \\ \vdots \\ \theta_{k0} \end{pmatrix} + \begin{pmatrix} \theta_{11}^1 & \theta_{12}^1 & \dots & \theta_{1k}^1 \\ \\ \theta_{21}^1 & \theta_{22}^1 & \dots & \theta_{2k}^1 \\ \vdots & \vdots & & \vdots \\ \theta_{k1}^1 & \theta_{k2}^1 & \dots & \theta_{kk}^1 \end{pmatrix} \begin{pmatrix} \mathrm{w}_{1t-1} \\ \\ \mathrm{w}_{2t-1} \\ \vdots \\ \mathrm{w}_{kt-1} \end{pmatrix} +$$

$$\begin{pmatrix} \theta_{11}^2 & \theta_{12}^2 & \dots & \theta_{1k}^2 \\ \\ \theta_{21}^2 & \theta_{22}^2 & \dots & \theta_{2k}^2 \\ \vdots & & & \vdots \\ \theta_{k1}^2 & \theta_{k2}^2 & \dots & \theta_{kk}^2 \end{pmatrix} \begin{pmatrix} \mathrm{w}_{1t-2} \\ \\ \mathrm{w}_{2t-2} \\ \vdots \\ \mathrm{w}_{kt-2} \end{pmatrix} + \cdots +$$

$$\begin{pmatrix} \theta_{11}^{p} & \theta_{12}^{p} & \dots & \theta_{1k}^{p} \\ \theta_{21}^{p} & \theta_{22}^{p} & \dots & \theta_{2k}^{p} \\ \vdots & \vdots & & \vdots \\ \theta_{k1}^{p} & \theta_{k2}^{p} & \dots & \theta_{kk}^{p} \end{pmatrix} \begin{pmatrix} \mathrm{w}_{1t-p} \\ \mathrm{w}_{2t-p} \\ \vdots \\ \mathrm{w}_{kt-p} \end{pmatrix} + \begin{pmatrix} \mathrm{a}_{1t} \\ \mathrm{a}_{2t} \\ \vdots \\ \mathrm{a}_{kt} \end{pmatrix}$$

VECTOR MOVING AVERAGE (VMA) MODEL

In this case, we will consider a VMA model of (2)-dimensional VMA (1) model, that is

$$w_t = \mu + a_t - \theta_1 a_{t-1} \tag{3.6}$$

We can also rewrite this, using $\theta_1 = [\theta_1]_{ij}$, therefore

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} - \begin{bmatrix} \theta_{1,11} & \theta_{1,12} \\ \theta_{1,21} & \theta_{1,22} \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2t-1} \end{bmatrix} \tag{3.7}$$

This is equivalent to the following equations

$$w_{1t} = \mu_1 + a_{1t} - \theta_{1,11} a_{1,t-1} - \theta_{1,12} a_{2t-1}$$

$$w_{2t} = \mu_2 + a_{2t} - \theta_{1,21} a_{1,t-1} - \theta_{1,22} a_{2t-1}$$

From the prior equations, the coefficient $\theta_{1,12}$ measures the impact of a_{2t-1} on the w_{1t} in the presence of the model, also $\theta_{1,21}$ present the effect of $a_{1,t-1}$ on w_{2t} in the midst of a_{2t-1}.

VECTOR AUTOREGRESSIVE MOVING AVERAGE (VARMA) MODEL

An N-dimensional variable w_t is a VARMA (p,q) model if a process.

$$\phi(B) w_t = \phi_0 + \theta(B) a_t \tag{3.8}$$

In which we defined, ϕ_0 as a constant vector, $\phi(B) = \mathrm{I}_k - \Sigma_{i=1}^{p} \phi_i B^1$ and $\theta(B) = \mathrm{I}_k - \Sigma_{i=1}^{q} \theta_i B^1$ are two matrix polynomials, and $\{ a_t \}$ is a sequence of independent and identically distributed random vectors with zero mean and positive covariance matrix E_a and B is the backward shift operator define as $B^k w_t = w_{t-k}$.

SEASONAL MODEL

The direct generalization of the univariate model of seasonal time w_t is written as;

$$(1-B)(1-B^S) z_t = (\mathrm{I}_k - \theta B)(\mathrm{I}_k - \Theta B^S) a_t \tag{3.9}$$

With this equation, we can also generate the multivariate model.

Therefore we refer this to the model as the seasonal model. We may also rewrite the equation (3.9) to be;

$$z_t = \frac{\mathrm{I}_k - \theta B}{1-B} X \frac{\mathrm{I}_k - \Theta B^S}{1-B^S} a_t \tag{3.10}$$

So by letting $w_t = (1-B)(1-B^s) z_t$ and $\Gamma_{w,e}$ be the lag autocovariance matrix of w_t. It's straight forward to see that $w_t = (\mathrm{I}_k - \theta B)(\mathrm{I}_k - \Theta B^S) a_t \Rightarrow (\mathrm{I}_k - \theta B - \Theta B^S + \theta \Theta B^{S+1}) a_t$ (3.11)

See [15]

ORDER OF SELECTION

In this work, we will emulate the proposed tool of [1], and that consisting of model specification, estimation, and diagnostic checking on multivariate analysis. We will use the recent procedure of [2]. But this approach of selecting the model order of multivariate time series was first proposed by [12]. Behind the approach is to compare different sets of the multivariate model that amount to examining the hypothesis of testing;

$H_0;\ \emptyset_\rho = 0$ Versus $H_\propto;\ \emptyset_\rho \neq 0$

INFORMATION CRITERIA

Models are successful in concluding any mathematical model based on the knowledge criteria. So, we understand that all parameters are based on chance, consist of two properties. Firstly, components are concerned with the model data's goodness of fit test, while the second component penalizes more complex models.

These methods are;

$$AIC(\ell) = In\,|\,\tilde{\mathrm{E}}_{a,\ell}\,| + \frac{2}{T}\ell K^2 \qquad (3.12)$$

$$BIC(\ell) = In\,|\,\tilde{\mathrm{E}}_{a,\ell}\,| + \frac{In(T)}{T}\ell K^2 \qquad (3.13)$$

MODEL CHECKING

Model-checking is a major aspect of the examination of the model; it is also known as a diagnostic test. In model design, this plays a significant role, such as multivariate normality, a typical model is said to be adequate.

Result

DATA

The data used in this work was collected from the National Bureau of Statistics (NBS), the data contain quarterly government records for sectors GDP of Agric, Industries, B/ construction, W/Retail, and Services growth rate from 1985-2017, a total of 33 years.

VARIABILITY OF THE VARIABLES

The observation of the variability with the series graph on figure 1,

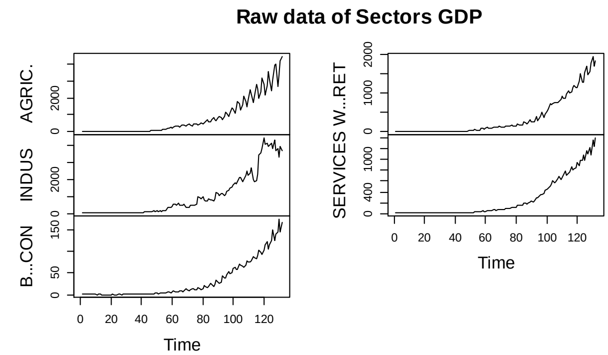

FIGURE 1; PLOT OF THE RAW DATA

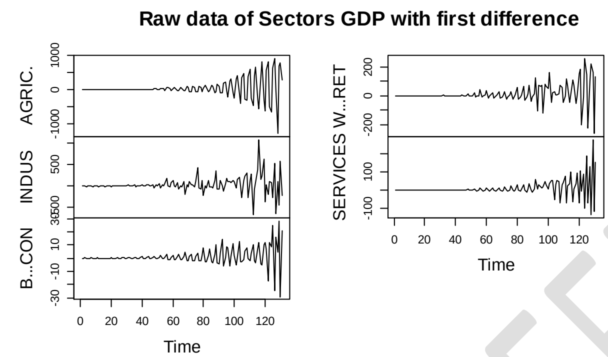

FIGURE 2; PLOT OF FIRST DIFFERENCE OF THE RAW DATA

The variability statistics of the five variables in figure 1 indicate that there is a seasonality trend in all the variables. In particular, the graph of building and construction shows linearity changing without limits, together with that of services, wholesale, and retails. While that of industries and agriculture abrupt start and permanent effect of linearity were also indicated. In all the variable graphs, we observed that the effect of the trend started in 2004 that was when the GDP of these sectors started to experience challenges or shock on the economic market. We all also noticed that the graph in figure 1 is not for stationary series, so to make it to be stationary we plot the graph of the first difference, which will help us to indicate the seasonality trend. So we difference the data series (once) to obtain stationary series and identify the season order (see figure 2)

Table 1; MODEL SELECTION USING AIC AND BIC CRITERIA

S/N	VAR –ORDER	AIC	BIC
1	SVARMA(4,1,1)(1,0,0)$_{12}$	-26.789	-23.837
2	SVARMA(3,1,0)(1,1,0)$_{4}$	-23.267	-21.0274
3	SVARMA(2,1,0)(1,0,0)$_{4}$	-24.2535	-21.617
4	SVARMA(3,1,0)(1,0,0)$_{12}$	-25.9821	-22.6453
5	SVARMA(3,1,0)(1,0,0)$_{4}$	-25.1474	-22.5869
6	SVARMA(4,1,0)(1,1,0)$_{12}$	-25.2186	-22.1894
7	SVARMA(4,1,1)(1,0,0)$_{3}$	-25.817	-22.0269
8	SVARMA(4,1,1)(1,0,0)$_{6}$	-25.8361	-22.2874
9	SVARMA(4,1,0)(2,0,0)$_{12}$	-25.0187	-22.0994
10	SVARMA(4,1,0)(1,0,0)$_{12}$	-26.3964	-23.4431

In the order of selecting the model in table 1, K=5, the parameters of the model (no. of variables used in the work). P= SVARMA (p,d,q)(P,D,Q)$_{S}$ (lag); which stands for the vector order of the model. T=132, stand for the sample size. On applying the sequential likelihood ratio test, using the information criteria on the data. We subjected the data into SVARMA (p,d,q)(P,D,Q)$_{S}$ (lag) models, in the order; we selected the best model in table 1. We observed that the order selected by AIC, and BIC, of the SVARMA (4,1,1)(1,0,0)$_{12}$ model, have the least value of AIC, and BIC. So statistically speaking, the SVARMA (4,1,1)(1,0,0)$_{12}$ model of the GDP order of selection is the best in modeling the data GDP in Nigeria.

MODEL PRESENTATION OF THE FITTED SVARMA (4, 1, 1) (1, 0, 0)$_{12}$ MODEL

The fitted SVARMA(4,1,1)(1,0,0)$_{12}$ model of the logarithm growth rates of the quarterly sectors GDP of agriculture, industries, building &construction, wholesale & retails, and services in Nigeria can be presented as follows;

$$Z_t = \begin{bmatrix} 0.0097 \\ 0.0030 \\ 0.0351 \\ 0.0158 \\ 0.0124 \end{bmatrix} + \begin{bmatrix} -0.0713 & -0.0084 & 0.0742 & 0.01269 & -0.3112 \\ 1.2032 & -0.3159 & 0.5776 & -0.6418 & -0.1086 \\ -0.0903 & -0.1084 & -0.0832 & 0.0370 & -0.0702 \\ 0.1599 & 0.1521 & -0.1680 & -0.1616 & -0.0637 \\ 0.0537 & 0.0379 & -0.0841 & -0.0048 & -0.0676 \end{bmatrix} Z_{t-1}$$

$$+ \begin{bmatrix} -0.1405 & 0.0244 & 0.2240 & 0.10680 & -0.2285 \\ 0.5970 & -0.0834 & -0.1380 & -0.4973 & 0.0180 \\ -0.0478 & -0.0192 & -0.1190 & -0.0290 & 0.0348 \\ 0.1477 & 0.0569 & 0.2830 & -0.0997 & -0.4033 \\ -0.0021 & 0.0266 & 0.1090 & -0.0223 & -0.1791 \end{bmatrix} Z_{t-2}$$

$$+ \begin{bmatrix} -0.0170 & -0.0036 & 0.0876 & 0.0972 & -0.1408 \\ 0.6349 & -0.1894 & 0.0576 & -0.6138 & 0.6543 \\ 0.0233 & -0.0199 & -0.0346 & 0.0778 & -0.1380 \\ 0.1566 & 0.0078 & 0.0734 & -0.1367 & 0.0235 \\ 0.1182 & -0.0077 & 0.0658 & 0.0313 & -0.2295 \end{bmatrix} Z_{t-3}$$

$$
+\begin{bmatrix} 0.6408 & -0.0497 & 0.1350 & -0.0190 & 0.0847 \\ 0.2791 & 0.0932 & 0.4450 & -0.5166 & -0.0986 \\ -0.1913 & -0.0908 & 0.6510 & 0.0876 & 0.1881 \\ 0.0492 & -0.0402 & 0.4260 & 0.5966 & -0.1747 \\ 0.0775 & -0.0804 & 0.2700 & -0.0460 & 0.6085 \end{bmatrix} Z_{t-4}
$$

$$
+\begin{bmatrix} -0.02426 & -0.03012 & -0.0677 & -0.1535 & 0.3541 \\ -0.03301 & 0.16157 & -0.7219 & -0.2516 & 0.8880 \\ -0.04015 & 0.01056 & 0.3094 & 0.0139 & -0.0893 \\ 0.00385 & 0.00166 & 0.3018 & -0.1221 & -0.0123 \\ -0.08460 & 0.02527 & 0.5310 & -0.1652 & 0.1129 \end{bmatrix} Z_{t-1,12}
$$

$$
\begin{bmatrix} 0.2833 & -0.0275 & 0.0288 & -0.0683 & -0.300 \\ 1.111 & -0.2316 & 0.5661 & -0.4788 & 0.081 \\ -0.1269 & -0.1302 & 0.1964 & 0.0589 & -0.1450 \\ -0.1637 & 0.1922 & 0.3205 & 0.3656 & -0.740 \\ 0.0706 & 0.0406 & 0.1348 & 0.121 & -0.172 \end{bmatrix} a_{t-1}
$$

And the residual covariance matrix is

$$
\Sigma_a = \begin{bmatrix} 0.00382 & 0.00308 & 0.00144 & 0.00316 & 0.00164 \\ 0.00308 & 0.003028 & 0.001296 & 0.002977 & 0.003225 \\ 0.00144 & 0.001296 & 0.00264 & 0.001285 & 0.001589 \\ 0.00316 & 0.002977 & 0.001285 & 0.005377 & 0.001699 \\ 0.00164 & 0.003225 & 0.001589 & 0.001699 & 0.001792 \end{bmatrix}
$$

Similarly, we can rewrite the above matrix model as the equation below;

$Z_{1t} = 0.0097 - 0.0713Z_{1,11,t-1} - 0.0084Z_{1,12,t-1} + 0.0742Z_{1,13,t-1} +$
$0.01269Z_{1,14,t-1} - 0.3112Z_{1,15,t-1} - 0.1405Z_{2,11,t-2} + 0.0244Z_{2,12,t-2} + 0.2240Z_{2,13,t-2} +$
$0.10680Z_{2,14,t-2} - 0.2285Z_{2,15,t-2} - 0.0170Z_{3,11,t-3} - 0.0036Z_{3,12,t-3} + 0.0876Z_{3,13,t-3} +$
$0.0972Z_{3,14,t-3} - 0.1408Z_{3,15,t-3} + 0.6408Z_{4,11,t-4} - 0.0497Z_{4,12,t-4} + 0.1350Z_{4,13,t-4} -$
$0.0190Z_{4,14,t-4} + 0.0847Z_{4,15,t-4} - 0.02426Z_{12,11,t-12} - 0.03012Z_{12,12,t-12} +$
$0.0677Z_{12,13,t-12} - 0.1535Z_{12,14,t-12} + 0.3541Z_{12,15,t-12} + 0.2833a_{1,11,t-1} -$
$0.0275a_{1,12,t-1} + 0.0288a_{1,13,t-1} - 0.0683a_{1,14,t-1} - 0.3000a_{1,15,t-1}$

$Z_{2t} = 0.0030 + 1.2032Z_{1,21,t-1} - 0.3156Z_{1,22,t-1} + 0.5776Z_{1,23,t-1}$
$- 0.6418Z_{1,24,t-1} - 0.1086Z_{1,25,t-1} + 0.5970Z_{2,21,t-2} - 0.0834Z_{2,22,t-2} - 0.1380Z_{2,23,t-2} -$
$0.4973Z_{2,24,t-2} + 0.0180Z_{2,25,t-2} + 0.6349Z_{3,21,t-3} - 0.1894Z_{3,22,t-3} + 0.0576Z_{3,23,t-3} -$
$0.6138Z_{3,24,t-3} + 0.6543Z_{3,25,t-3} + 0.2791Z_{4,21,t-4} + 0.0932Z_{4,22,t-4} + 0.4450Z_{4,23,t-4} -$
$0.5166Z_{4,24,t-4} - 0.0986Z_{4,25,t-4} - 0.03301Z_{12,21,t-12} + 0.1616Z_{12,22,t-12} -$
$0.7219Z_{12,23,t-12} - 0.2516Z_{12,24,t-12} + 0.8880Z_{12,25,t-12} + 1.1115a_{1,21,t-1} -$
$0.02316a_{1,22,t-1} + 0.5661a_{1,23,t-1} - 0.4788a_{1,24,t-1} + 0.0810a_{1,25,t-1}$

$Z_{3t} = 0.0351 - 0.0903Z_{1,31,t-1} - 0.1084Z_{1,32,t-1} - 0.932Z_{1,33,t-1} +$
$0.0370Z_{1,34,t-1} - 0.0702Z_{1,35,t-1} - 0.0478Z_{2,31,t-2} - 0.0192Z_{2,32,t-2} - 0.1190Z_{2,33,t-2} -$
$0.0290Z_{2,34,t-2} + 0.0348Z_{2,35,t-2} + 0.0233Z_{3,31,t-3} - 0.0199Z_{3,32,t-3} - 0.0346Z_{3,33,t-3} +$
$0.0778Z_{3,34,t-3} - 0.1380Z_{3,35,t-3} - 0.1913Z_{4,31,t-4} - 0.0908Z_{4,32,t-4} + 0.6510Z_{4,33,t-4} +$
$0.0876Z_{4,34,t-4} + 0.1881Z_{4,35,t-4} - 0.0402Z_{12,31,t-12} + 0.0106Z_{12,32,t-12} +$

$0.3094Z_{12,33,t-12} + 0.0139Z_{12,34,t-12} - 0.0893Z_{12,35,t-12} - 0.1259a_{1,31,t-1} -$
$0.13026a_{1,32,t-1} + 0.1964a_{1,33,t-1} + 0.0589a_{1,34,t-1} - 0.1450a_{1,25,t-1}$

$$Z_{4t} = 0.0158 + 0.1599Z_{1,41,t-1} + 0.1521Z_{1,42,t-1} - 0.1680Z_{1,43,t-1} -$$

$0.1616Z_{1,44,t-1} - 0.0637Z_{1,45,t-1} + 0.1477Z_{2,41,t-2} + 0.0569Z_{2,42,t-2} + 0.0283Z_{2,43,t-2} -$
$0.0997Z_{2,44,t-2} - 0.4033Z_{2,45,t-2} + 0.1556Z_{3,41,t-3} - 0.0078Z_{3,42,t-3} - 0.0738Z_{3,43,t-3} -$
$0.1367Z_{3,44,t-3} + 0.0235Z_{3,45,t-3} + 0.0492Z_{4,41,t-4} - 0.0402Z_{4,42,t-4} + 0.4260Z_{4,43,t-4} +$
$0.5966Z_{4,44,t-4} - 0.1747Z_{4,45,t-4} + 0.00385Z_{12,41,t-12} + 0.00166Z_{12,42,t-12} +$
$0.3018Z_{12,43,t-12} - 0.1221Z_{12,44,t-12} - 0.0123Z_{12,45,t-12} - 0.1637a_{1,41,t-1} +$
$0.1922a_{1,42,t-1} + 0.32054a_{1,43,t-1} + 0.3656a_{1,44,t-1} - 0.7400a_{1,45,t-1}$

$$Z_{5t} = 0.0124 + 0.0537Z_{1,51,t-1} + 0.0379Z_{1,52,t-1} - 0.0841Z_{1,53,t-1} -$$

$0.0048Z_{1,54,t-1} - 0.676Z_{1,55,t-1} - 0.0021Z_{2,51,t-2} + 0.0266Z_{2,52,t-2} + 0.1090Z_{2,53,t-2} -$
$0.0223Z_{2,54,t-2} - 0.1791Z_{2,55,t-2} + 0.1182Z_{3,51,t-3} - 0.0078Z_{3,52,t-3} + 0.0658Z_{3,53,t-3} +$
$0.0313Z_{3,54,t-3} - 0.2295Z_{3,55,t-3} + 0.0775Z_{4,51,t-4} - 0.0804Z_{4,52,t-4} + 0.2700Z_{4,53,t-4} -$
$0.0460Z_{4,54,t-4} + 0.6085Z_{4,55,t-4} - 0.0846Z_{12,51,t-12} + 0.2527Z_{12,52,t-12} +$
$0.5310Z_{12,53,t-12} - 0.1652Z_{12,54,t-12} + 0.1129Z_{12,55,t-12} + 0.0706a_{1,51,t-1} +$
$0.0406a_{1,52,t-1} + 0.1348a_{1,53,t-1} + 0.0121a_{1,54,t-1} - 0.172a_{1,55,t-1}$

Where Z_{1t}, Z_{2t}, Z_{3t}, Z_{4t}, and Z_{5t} are Agriculture, Industries, Building & Construction, Wholesale & Retail, and Services

The standard error of the coefficient estimates in the model showed that some of the standard error coefficient estimates and the residual on the parameters were not statistically significant at the 5% level. Hence we have to carry out model simplification by removal of the insignificant statistical coefficient of the estimated parameters.

MODEL CHECKING OF THE RESIDUAL ON SIMPLIFIED SVARMA (4,1,1)(1,0,0)$_{12}$

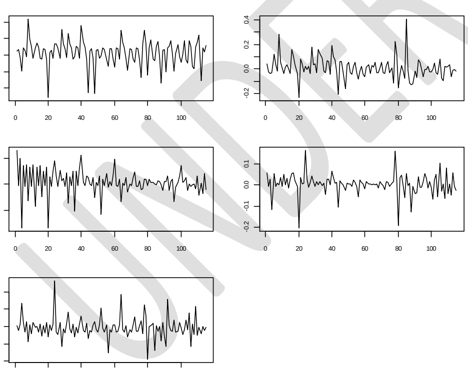

FIGURE 3 RESIDUAL PLOTS OF THE SIMPLIFIED SVARMA (4,1,1)(1,0,0)$_{12}$ MODEL FOR THE SECTORS GDP OF AGRICULTURE, INDUSTRIES, BUILDING & CONSTRUCTION, WHOLESALE & RETAILS AND SERVICES IN NIGERIA

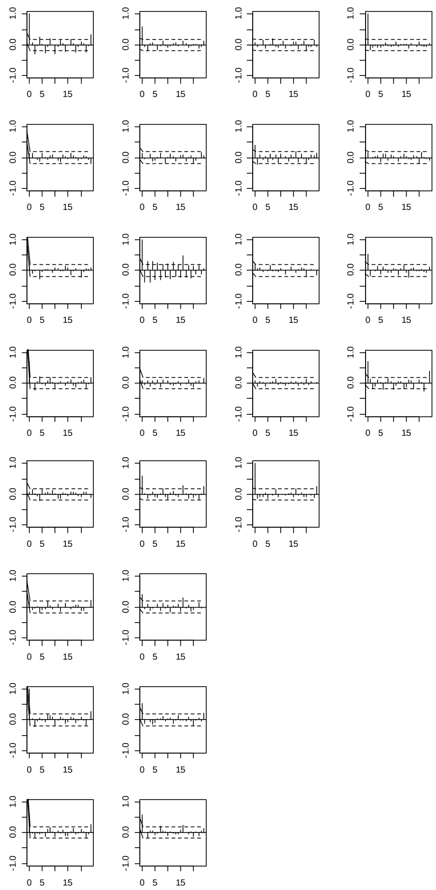

FIGURE 4 RESIDUAL CROSS CORRELATION MATRICES OF THE SIMPLIFIED SVARMA (4,1,1)(1,0,0)$_{12}$ MODEL FOR THE SECTORS GDP OF AGRICULTURE, INDUSTRIES, BUILDING & CONSTRUCTION, WHOLESALE & RETAILS AND SERVICES IN NIGERIA

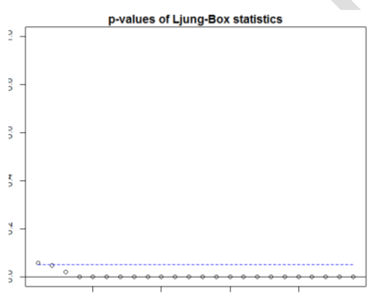

FIGURE 5; PLOT OF LJUNG-BOX STATISTICS OF THE RESIDUAL OF THE SIMPLIFIED SVARMA (4,1,1)(1,0,0)$_{12}$ MODEL FOR THE SECTORS GDP OF AGRICULTURE, INDUSTRIES, BUILDING & CONSTRUCTION, WHOLESALE & RETAILS AND SERVICES IN NIGERIA

From the plots of the simplified SVARMA (4, 1, 1) (1, 0, 0)$_{12}$, the residual of the series plot indicates that the model is of the goodness of fit. Whereas figure 4 also shows the residual cross-correlation matrices, as we can see, the dashed lines of the serials correlations indicate the approximate 2 standard error limits of the correlations. That is $\pm 2/\sqrt{T}$. Based on the serial correlation matrices, we now conclude that the residual of the model has no strong serial correlation. The plot of the P-values of the $Q_k(m)$ statistics applied to the residual of the simplified model also confirmed that the model is fitted. So in conclusion, we agreed that the simplified SVARMA(4,1,1)$(1,0,0)_{12}$ is adequate for the model of GDP of agriculture, industries, building & construction, wholesale & retails, and services in Nigeria.

Table 2; Forecasts, Standard Error and Root Means Square of quarterly sectors GDP, in logarithm, for Agricultures, Industries, Building $ Construction, wholesale $ Retail, and Services using SIMPLIFIED SVARMA (4,1,1)$(1,0,0)_{12}$ model

Forecasts

	AGRIC.	INDUS	B$CON	W$RET	SERVICES
[1,]	8.053	8.439	5.155	7.560	7.116
[2,]	8.230	8.305	5.315	7.792	7.377
[3,]	8.422	8.583	5.084	7.598	7.284
[4,]	8.460	8.518	5.190	7.713	7.430
[5,]	8.154	8.800	5.227	7.700	7.283
[6,]	8.326	8.502	5.411	7.967	7.532
[7,]	8.500	8.705	5.164	7.709	7.407
[8,]	8.560	8.529	5.268	7.834	7.554
[9,]	8.251	8.835	5.293	7.780	7.404
[10,]	8.420	8.485	5.494	8.074	7.653
[11,]	8.575	8.720	5.240	7.776	7.512
[12,]	8.645	8.525	5.343	7.917	7.660

Standard Errors of predictions

0.083	0.1670	0.049	0.0724	0.046
0.102	0.201	0.0614	0.0897	0.0598
0.114	0.222	0.0674	0.1033	0.0677
0.128	0.233	0.0714	0.1178	0.0718
0.153	0.257	0.0895	0.1519	0.0940
0.177	0.273	0.0964	0.1689	0.1048
0.192	0.284	0.1006	0.184	0.1128
0.206	0.298	0.104	0.200	0.1178
0.226	0.305	0.116	0.224	0.1340
0.244	0.315	0.1207	0.2382	0.1431
0.257	0.324	0.1245	0.2507	0.1509
0.268	0.334	0.1277	0.2632	0.1567

Root MSE of predictions

0.093	0.186	0.0551	0.08052	0.051
10.994	20.79	6.684	9.784	7.0636
9.376	17.41	5.136	9.429	5.84
10.53	13.24	4.31	10.46	4.420
15.64	20.00	9.96	17.68	11.18
16.20	16.94	6.605	13.62	8.57
13.87	14.38	5.34	13.68	7.67
13.63	16.34	4.83	14.25	6.308
17.24	12.11	9.50	18.91	11.77
16.84	14.98	6.11	14.54	9.230
15.25	14.028	5.65	14.42	8.850
13.89	14.56	5.210	14.78	7.81

Discussion

Variability (graphical) presentations of the variables were carried out on the variables of the model to confirm the seasonality trend. We also adopt the first different process to obtain the seasonal order of the model. By adoption of information criteria on the data in order of selecting the model in table 1, we observed that the order selected by AIC, and BIC, of the SVARMA $(4,1,1)(1,0,0)_{12}$ model, have the least value of AIC, and BIC. However, we select the SVARMA $(4, 1, 1)(1,0,0)_{12}$ model as the best in modeling the data GDP in Nigeria. We also examine one-step to twelve-step (3 years) ahead projections of the rates at the projected origin 2017. 12. 12. We include the standard errors and the predictions' root mean square error. We found from the results tables that the forecast point of the five series moves close to sample means of increasing data at the forecast horizon, which indicates the proof of reverting means as we predicted. Secondly, with the horizon, the Standard Error of Prediction and the root MSE of the predictions increase. So this is reasonable because a stationary SVARMA $(4, 1, 1)(1, 0, 0)_{12}$ is a mean-reverting series with the fact that, there exists long-term stability in the variables. The root means square error and the standard error of the forecast in table 2 can also be used to construct interval predictions. For instance, a five-step 95% interval for the Agriculture GDP is $0.153 \pm 1.96X15.64$ and $0.153 \pm 1.96 + 11.18$ respectively.

Summary

The "Seasonal Vector Autoregressive Moving Average" was used to analyze the Gross Domestic Product (GDP) growth rate of five (5) sectors in this research: Agriculture, Industries, Building/Construction, Wholesales/Retails, and Services. The data was gathered from the National Bureau of Statistics and spans 33 years, from 1985 to 2017. To evaluate the model, Real (R) software was used. The five variables' time series plots reveal that they all have a seasonality pattern that is not stationary. We difference the data series (once) to obtain stationary series and describe the seasonal order to show the seasonality pattern in order to obtain seasonality. The best model and Lag's were chosen using Akaike Information Criteria in this study; based on the information criteria in the results, the SVARMA (4, 1, 1) (1, 0, 0)12 is the best model selected. The results show that the rate of growth in the Agriculture sector is slowing, the rate of growth in the Industries sector is slowly decreasing, the rate of growth in the Building/Construction sector is rising, the rate of growth in the Wholesales/Retails sector is not stable, and the rate of growth in the services sector is weak in Nigeria.

References

[1]Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (2008), Time Series Analysis, Forecasting and Control. *John Willey and Sons, Inc. Hoboken, New Jersey. ISBN978-0-470-27284-8 (cloth). QA280.B67 2008. 519.5_5—dc22*

[2]Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (2008), Time-series Analysis; Forecasting and Control, Revised ed., *Holden day, San Francisco, CA.*

[3]Fadli F. A , Zurida M., Teh S. A, Norazidah S., Nurul S. B. and Kamaruzaman J. (2011).

Multivariate time series analysis on the correlation between inflation rate and Employment Rate with Gross Domestic Products, *World Applied Science Journal, 61-66.*
[4]Hair. Jr. J.F., Willian Black, Barry J. Babin and R.E. Anderson(1998), Multivariate Data Analysis With Readings. *Pearson Prentice Hall, Issn:00401706.*
[5]Lin H. –C. and Xiraxagar S. (2006), Seasonality of Hip Fractures and Estimates of Season-attributable effects: A Multivariate ARIMA Analysis of Population-based Data, *International Osteoporois Foundation and National Osteoposis foundation, 17: 795-806.*
[6]Mark T. L., Hazem D, and An-sing C. (2000) forecasting Stock Indices; a comparison of classification and level estimation models, *International Journal of Forecasting 16, 173-190.*
[7]Oluwatomi, O. (2011), http://www.tribune.com.ng/inder.php/wealth.
[8]Philip Hans France (1994), A Multivariate Approach to Modeling Univariate Seasonal Time -series *Journal of Econometric 63 133-151.*
[9]Quintana J. and West M. (2009). An alternative model for multiple time series. *European Scientific Journal; Vol 9, No 4. ISSN: 1857- 7881(print)e-ISSN-7431.*
[10]Rapach D. E., and Strauss J. (2008), Structural breaks and GARCH models of Exchanges rate volatility. *Journal of Applied Econometrics, vol. 23, issue 1, 65-90*
[11]Ruiz F., Gonzalez- Realadoa M. L., and Munoz J.M . (1997), Multivariate Analysis Applied to Total and Living Fauna: seasonal Ecology of recent Benthic Ostracode of the North Cadiz Gulf Coast (Southwestern Spain). *Elsevier Science B.V, Marine Micropaleontology 31: 183-203.*
[12]Tiao, G. C. and Box, G. E. P. (1981). Modeling multiple time series with applications. *Journal of the American Statistical Association*, **76**: 802–816.
[13]Tsay, R. S. (1986), Nonlinearity Test for Time Series; *Biomettica 73,461-466*.
[14]Tsay, R. S. (2010). Analysis of Financial Time Series. *3rd Edition. John Wiley & Sons, Inc, Hoboken, NJ.*
[15]Tasy. R. (2013), Multivariate Time Series Analysis With R And Finance Application, *John Wiley and Son.*
[16]Zacharias Psaradkis and Marian Vavra (2014), Portmanteau Test for Linearity of Stationary Time series, *Econometric reviews, DOI:10.1080/0747938 (Birkbeck, Department Of Economics, Mathematics and Statistics*