Original Research Article

Primaries Oblateness effects on the Collinear Libration Points in the restricted-three body problem

Abstract

In this work, the canonical Hamiltonian form of the restricted three- body problem including the effects of primaries oblateness is presented. Moreover, the collinear libration points are obtained. In addition to this, the relation between position of libration points and variation in (mass ration , oblateness coefficients A1 and A2) is studied. The results obtained are a good agreement with Perdios [5] & Singh [8]. The Poincare surface section PSS is used to illustrate the stability of motion around each of the collinear libration points. A numerical application on the real system Earth-Moon is presented.

Key Words: restricted three body problem, oblateness effects, Poincare surface of section, stability motion, libration points.

Introduction

A special case of general three body problem is the restricted three body problem RTBP which play an important role in celestial mechanics. The equations of motion, in general case are non–linear in nature and it is more difficult to obtain analytic solutions for the general problem. Hence some restrictions were put to overcome this problem as the circular and elliptical RTBP. The restricted three-body problem with oblate primaries has also attracted the interest of many Researches [1-4]. Perdios [5] investigated the combined influence of the oblateness and radiation pressures of the primaries on collinear points moreover calculated Lyapunov planar and 3D family of periodic orbits around these points. Markello [4] obtained zero velocity and libration points for Hill's problem with the effect of oblateness. Abdul Raheem [6] found the periodic orbits around the triangular libration points under the effects of the centrifugal and the Coriolis forces together with solar radiation and oblateness of the two primaries. Singh [7] studied the nonlinear stability of the triangular points when both primaries are oblate spheroids. Ibrahim [9] obtained the Lissajous orbits and the phase spaces around collinear points under the effect of oblateness.

The perturbed mean motion n of the primaries is given by $n^2 = 1 + \frac{3}{2}(\mathrm{A1} + \mathrm{A2})$, where $Ai = \frac{r_{ei}^2 - r_{pi}^2}{5\,R^2}$ is the oblateness coefficient of m_1 and m_2 having the equatorial and polar radii as r_{ei} and r_{pi}, respectively and R is separation between the primaries. Then the effective potential equation with oblateness of two primaries is given by

$$V(x,y,z) = \frac{n^2}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)A1}{2r_1^3} + \frac{\mu\,A2}{2r_2^3} \qquad (1)$$

Where

$$r_1 = \sqrt{(\mathrm{x} - \mu)^2 + y^2} \qquad (2.1)$$

$$r_2 = \sqrt{(\mathrm{x} + 1 - \mu)^2 + y^2} \qquad (2.2)$$

The Hamiltonian system of R3BP with Oblateness

The Hamiltonian of the restricted three body problem with oblateness can be written as

$$\mathcal{H} = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) + yp_x - x\,p_y - \frac{n^2}{2}(x^2 + y^2) - \left[\frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)A1}{2r_1^3} + \frac{\mu\,A2}{2r_2^3}\right] \qquad (3)$$

The canonical form is given by

$$\dot{x} = \frac{\partial\mathcal{H}}{\partial p_x} = p_x + y \qquad (4.1)$$

$$\dot{y} = \frac{\partial\mathcal{H}}{\partial p_y} = p_y - x \qquad (4.2)$$

$$\dot{z} = \frac{\partial \mathcal{H}}{\partial p_z} = p_z \tag{4.3}$$

$$\dot{p}_x = -\frac{\partial \mathcal{H}}{\partial x} = -p_y - n^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu\ (x-\mu+1)}{r_2^3} - \frac{3A1\,(1-\mu)\,(x-\mu)}{2r_1^5} - \frac{3\ A2\ \mu\ (x-\mu+1)}{2\ r_2^5} \tag{4.4}$$

$$\dot{p}_y = -\frac{\partial \mathcal{H}}{\partial y} = p_x - n^2 y - \frac{(1-\mu).\ y}{r_1^3} - \frac{\mu\ y}{r_2^3} - \frac{3\ A1\ (1-\mu)y}{2\ r_1^5} - \frac{3\ A2\ \mu\ y}{2\ r_2^5} \tag{4.5}$$

$$\dot{p}_z = -\frac{\partial \mathcal{H}}{\partial z} = -\frac{(1-\mu).\ z}{r_1^3} - \frac{\mu\, z}{r_2^3} - \frac{3\ A1\ (1-\mu)\ .\ z}{2\ r_1^5} - \frac{3\ A2\ \mu\, z}{r_2^3} \tag{4.6}$$

Equations (4) represent the equation of motion of the third body under the effect of gravitational forces and the Oblateness of two primaries.

Location of the Libration points with effect of Oblateness

To obtain the location of libration points, put

$p_x = p_y = p_z = \dot{p}_x = \dot{p_y} = \dot{p_z} = 0$, then eqns. (4.4), (4.5) and (4.6) will be

$$-n^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu\ (x-\mu+1)}{r_2^3} - \frac{3A1\,(1-\mu)(x-\mu)}{2r_1^5} - \frac{3\ A2\ \mu\ (x-\mu+1)}{2\ r_2^5} = 0 \tag{5.1}$$

$$-n^2 y - \frac{(1-\mu).\ y}{r_1^3} - \frac{\mu\ y}{r_2^3} - \frac{3\ A1\ (1-\mu)y}{2\ r_1^5} - \frac{3\ A2\ \mu\ y}{2\ r_2^5} = 0 \tag{5.2}$$

$$-\frac{(1-\mu).\ z}{r_1^3} - \frac{\mu\, z}{r_2^3} - \frac{3\ A1\ (1-\mu)\ .\ z}{2\ r_1^5} - \frac{3\ A2\ \mu\, z}{r_2^3} = 0 \tag{5.3}$$

The collinear points can be determined from equation (5.1).

$$n^2 x = \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu\ (x+\mu-1)}{r_2^3} - \frac{3A1\,(1-\mu)(x+\mu)}{2r_1^5} - \frac{3\ A2\ \mu\ (x+\mu-1)}{2\ r_2^5} \tag{6}$$

The locations of the collinear points are

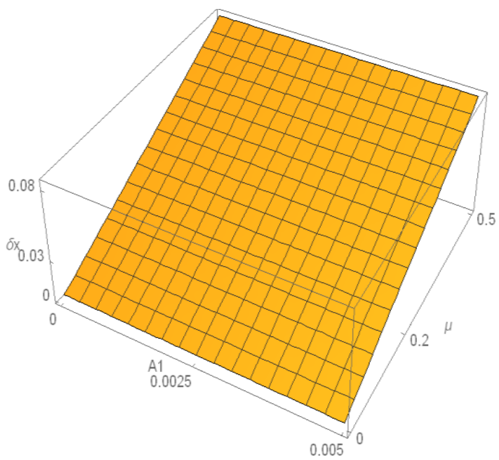
$$x_1 = \mu - 1 - \xi_1, \qquad x_2 = \mu - 1 + \xi_2\ , \qquad x_3 = \mu + \xi_3.$$

whereξ_1, ξ_2 and ξ_3 satisfy ninth degree polynomials while in Ibrahim [10] it was seventh degree when one of primary has oblate spheroid.

$$X1^9(2+3A1+3A2)+X1^8(10+15A1+15A2-2\mu-3A1\mu-3A2\mu)+X1^7(20+30A1+30A2-8\mu-12A1\mu-12A2\mu)+X1^6(18+30A1+30A2-8\mu-18A1\mu-18A2\mu)+X1^5(6+15A1+15A2+4\mu-12A1\mu-12A2\mu)+X1^4(6A1+3A2+12\mu-6A1\mu)+X1^3(8\mu+12A2\mu)+X1^2(2\mu+18A2\mu)+X1(12A2\mu)+3A2\mu=0 \quad (7)$$

$$(2+3A1+3A2)X2^9+(-10-15A1-15A2+2\mu+3A1\mu+3A2\mu)X2^8+(20+30A1+30A2-8\mu-12A1\mu-12A2\mu)X2^7+(-18-30A1-30A2+8\mu+18A1\mu+18A2\mu)X2^6+(6+15A1+15A2+4\mu-12A1\mu-12A2\mu)X2^5+(-6A1-3A2-12\mu+6A1\mu)X2^4+(8\mu+12A2\mu)X2^3+(-2\mu-18A2\mu)X2^2+12A2x\mu X2-3A2\mu=0 \quad (8)$$

$$(2+3A1+3A2)X3^9+(8+12A1+12A2+2\mu+3A1\mu+3A2\mu)X3^8+(12+18A1+18A2+8\mu+12A1\mu+12A2\mu)X3^7+(10+12A1+12A2+8\mu+18A1\mu+18A2\mu)X3^6+(10+3A1+3A2-4\mu+12A1\mu+12A2\mu)X3^5+(12-3A1-12\mu+6A1\mu)X3^4+(8-12A1-8\mu+12A1\mu)X3^3+(2-18A1-2\mu+18A1\mu)X3^2+(-12A1+12A1\mu)X3-3A1+3A1\mu=0 \quad (9)$$



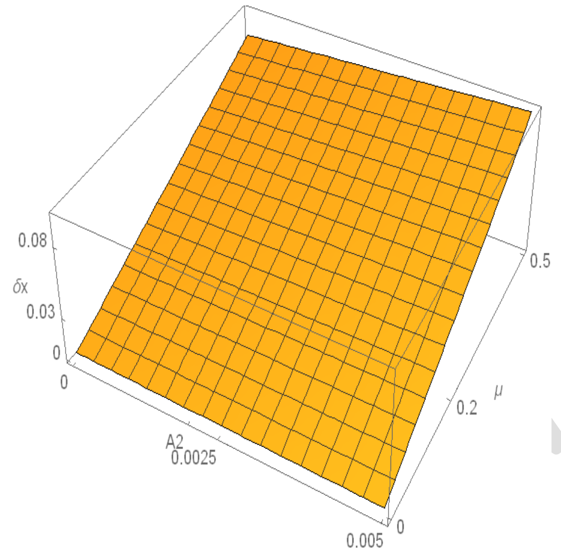
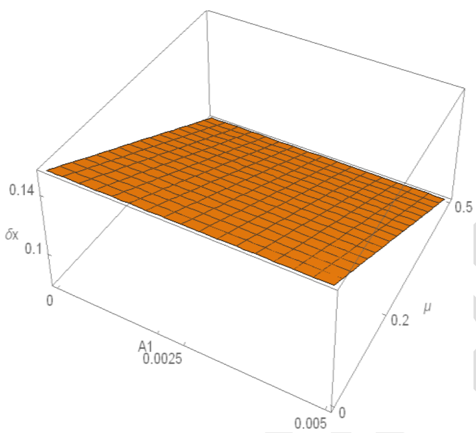


fig 1.a variation of L1 under the effect of A1

fig 1.b variation of L1 under the effect of A2



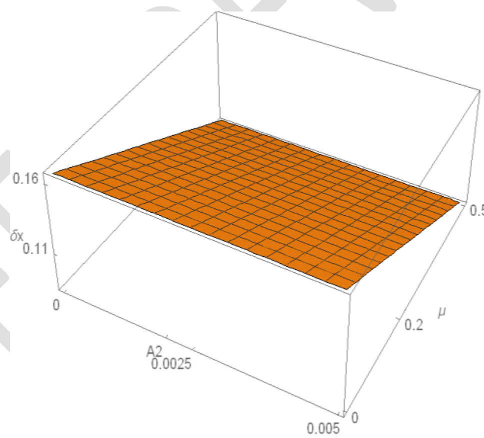
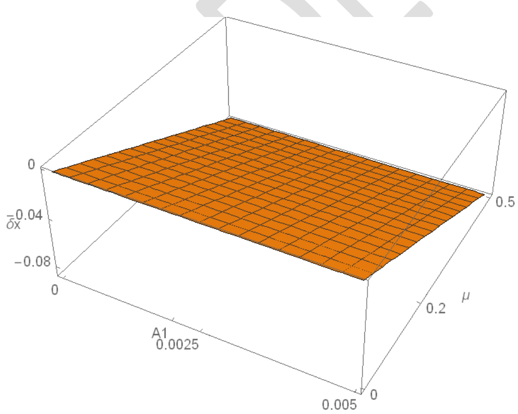


fig 2.a variation of L2 under the effect of A1

fig 2.b variation of L2 under the effect of A2



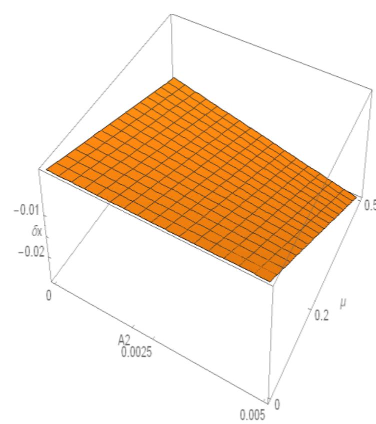


fig 3.a variation of L3 under the effect of A1

fig 3.b variation of L3 under the effect of A2

Figures 1–3 show surface representations of the variation of collinear libration (L1, L2, L3) under the effects of oblateness for the bigger and smaller primaries where μ ∈ (0,.5], A1 and A2 ∈ [0, 0.005]. Where fig 1 and fig 3 represent the increase in variation of L1 and L3 when μ, A1 and A2 increase contrary with fig2 the decrease in variation of L2 when μ, A1 and A2 increase. this coincide with [1], [2].

Poincare surfaces of section

In the restricted three-body problem, Poincare surface of section (PSS) is a powerful technique for studying stability of orbits which enables finding stable periodic and quasiperiodic around the two primaries. The four-dimensional phase space $(x, y, \dot{x}, \dot{y})$ is used to obtain (PSS) of the infinitesimal body at any instant. This is a good tool to study the stability of nature system which enables determine the regular or chaotic nature of the trajectory. Figures (4) and (5) show Poincare surface section, when $A1 = A2 = 0$ and $A1 \neq 0, A2 \neq 0$ for Earth-Moon system. The numerous islands can be observed which means that the behavior of the trajectory is likely to be regular, where the curves shrink down to a point, it represents a periodic orbit. To obtain this PSS an initial condition of x varies from 0.6 to 0.8 with δx = 0.001are used. So that, the canonical equations of motions (4) are integrated truncated up to 1000 steps using Runge–Kutta fixed step sized method.

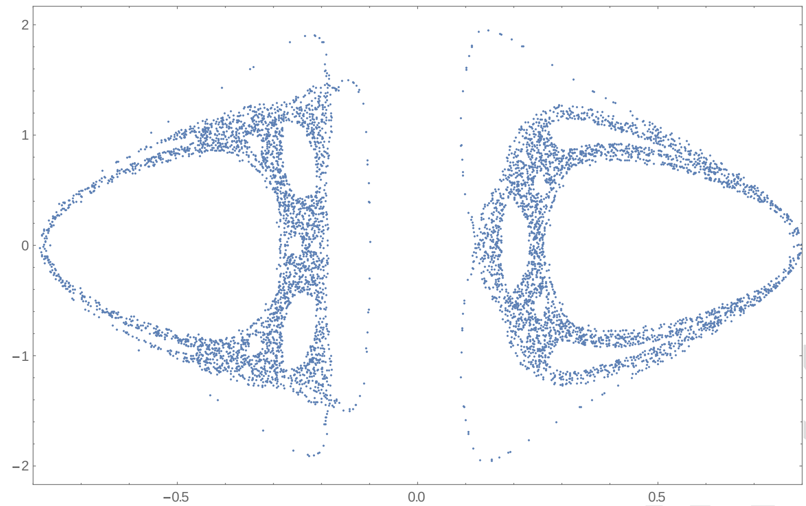


fig. 4 Poincare surface sections for earth-moon system A1= A2 = 0

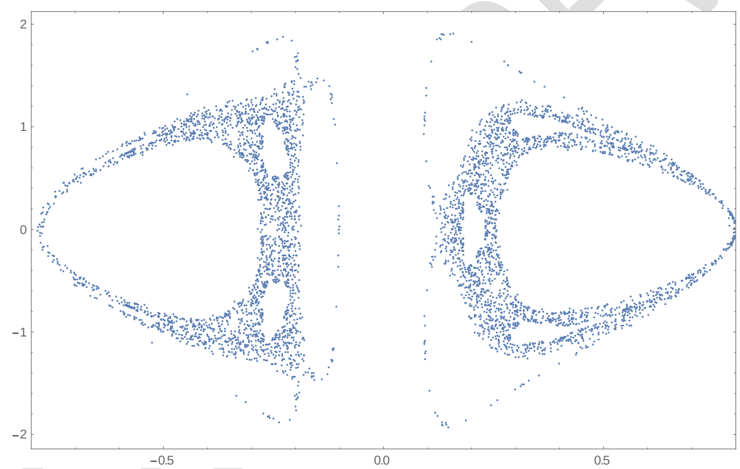
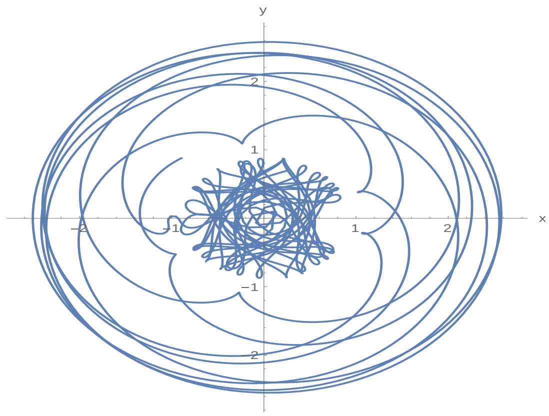


Fig. 5 Poincare surface sections for earth-moon system A1≠A2 ≠0



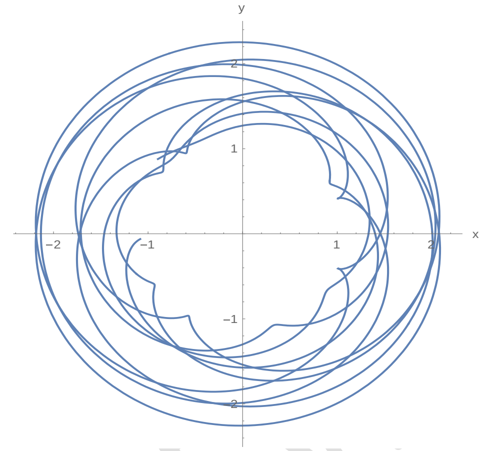
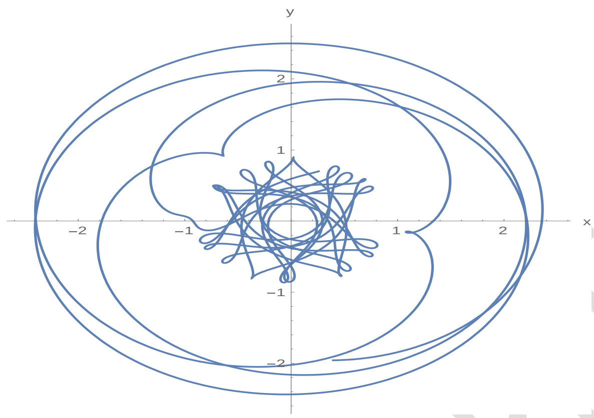


Figure 6.a Time from 0 to 200 figure 6.b Time from 200 to 324



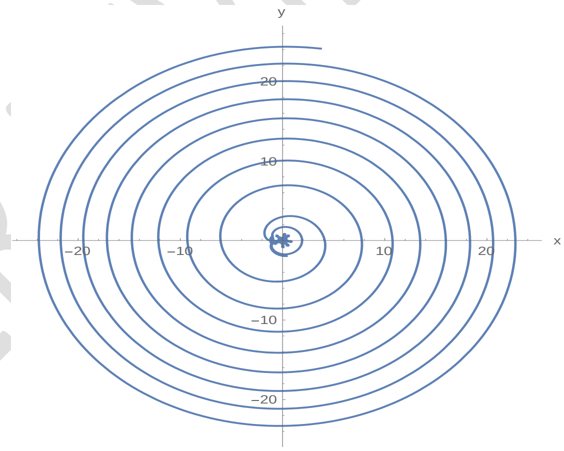


figure 7.a t from 0 to 100 figure 7.b t from 100 to 200

Figs.7, and Figs.8 represent the orbits of the third body. Figs.7 displays the orbit in presence of oblateness effect while Figs.8 displays orbit in absence of oblateness effect. The orbit of the infinitesimal body represents in first frame when $0 \leq t \leq 200$ whereas second frame when $200 \leq t \leq 340$. In the second frame, in the case of absence of oblateness effect, the orbit as spiral shape. However, with effect of oblateness, orbit becomes regular when $0 \leq t \leq 200$ which is shown in third frame while fourth frame shows the orbit when $100 \leq t \leq 200$.

Conclusion

This study related to the motion of the third body under the effects of the oblateness of the two primaries. It is found that the positions of L2 decreased under the effects of oblateness while the positions of L1 and L3 are increased under this effect. A PSS shows the stability of nature system which are a regular motion and a chaotic motion for the system without oblateness and under the effects of oblateness. These kinds of study are very important for the space missions. In near future the study will truncated under the effects of solar radiation pressure and the oblateness which is very important for the motion of solar sails.

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Orbital Motion Around the Collinear Libration Points of the Restricted Three-Body Problem, Journal of Advances in Mathematics and Computer Science, (2018) 29(1),1-16