Original Research Article

STATISTICAL ANALYSIS OF MOTHER-TO-CHILD HIV TRANSMISSION RATE USING A WEIBULL-EXPONENTIAL INVERSE EXPONENTIAL DISTRIBUTION

Abstract:

Distribution functions, their properties and interrelationships play a significant role in modeling naturally occurring phenomena. Numerous standard distributions have been extensively used over the past decades for modeling data in several fields, however, generalizing these standard distributions has produced several compound distributions that are more flexible compared to the baseline distributions. Acquired immune deficiency syndrome is a disease caused by human immunodeficiency virus (HIV) that leads to a continuous decay of the human body immune system. Over the past few years, the rate of mother-to-child transmission of HIV has been on a non-decreasing trend in Nigeria and hence becoming a threat to the health of the nation. The Weibull generalized family of distributions has been efficient in developing new continuous probability distributions with additional two shape parameters. In this paper, a Weibull-based model has been proposed and it is called “a Weibull-Exponential Inverse Exponential distribution”. The properties, estimation of parameters and application of the new distribution are presented and discussed in this paper. Adequate application and investigation of the new model was done using a dataset on the rate of mother-to-child transmission of HIV and the result was compared with that of other competing models.

Keywords: Weibull-G family, Weibull-Exponential Inverse Exponential Distribution, Properties, Maximum Likelihood Estimation, Application.

1 Introduction

Recently, numerous extended or compound probability distributions have been proposed in the literature for modeling real life situations and these compound distributions are found to be skewed, flexible and more better in statistical modeling compared to their standard counterparts ([1]- [14]).

In the year 2017, [15] developed an exponential inverse exponential distribution (EIED) with two parameters (a shape and scale parameter). This distribution was found to be better than the exponential and inverse exponential distribution with a study of its important mathematical and statistical properties, maximum likelihood estimation of parameters and applications using real life datasets can be found in [15]. It was also found to be skewed and flexible with an increasing hazard rate and different shapes.

The probability density function (pdf) of the Exponential Inverse Exponential distribution (EIED) according to [15] is defined by

$$g(x) = \frac{\theta\lambda}{x^2} \frac{e^{-\frac{\lambda}{x}}}{\left[1 - e^{-\frac{\lambda}{x}}\right]^2} e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \qquad (1)$$

The corresponding cumulative distribution function (cdf) of Exponential Inverse Exponential distribution (EIED) is given by

$$G(x) = 1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \qquad (2)$$

where, $x > 0, \lambda > 0, \theta > 0$; θ is the shape parameter and λ is a scale parameter.

Similarly, other generalizations of the inverse exponential distribution in the literature are; the odd Lindley inverse exponential distribution [16], the Kumaraswamy Inverse Exponential distribution [17], the exponentiated generalized Inverse Exponential distribution [18], a new Lindley-Exponential distribution [19], the transmuted odd generalized exponential-exponential distribution [20], the transmuted exponential distribution [21], transmuted inverse exponential distribution [22], the odd generalized exponential-exponential distribution [23], and the Weibull-exponential distribution [24].

Therefore, the aim of this article is to introduce a new continuous distribution called the Weibull-Exponential Inverse Exponential distribution (WEIED) using a method of generating continuous probability distributions proposed by [7].

The rest of this paper is organized in sections as follows: the newly proposed distribution is defined with its plots in section 2. Section 3 presents statistical properties of the new distribution. Section 4 looks at the estimation of parameters using maximum likelihood estimation (MLE). An application of the newly proposed model with other existing distributions to mother-to-child HIV transmission rate presented in section 5 and the final summary and conclusion is provided in section 6.

2. The Weibull-Exponential Inverse Exponential distribution (WEIED)

In this section, we have defined the cdf and pdf of the Weibull-exponential inverse exponential distribution using the method proposed by [7]. According to [7], the formula for deriving the cdf and pdf of any Weibull-based continuous distribution is defined as:

$$F(x) = \int_{0}^{-\log\left[G(x)\right]} \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} dt = e^{-\alpha\left\{-\log\left[G(x)\right]\right\}^{\beta}} \qquad (3)$$

and

$$f(x) = \alpha\beta \frac{g(x)}{G(x)} \left\{-\log\left[G(x)\right]\right\}^{\beta-1} e^{-\alpha\left\{-\log\left[G(x)\right]\right\}^{\beta}} \qquad (4)$$

respectively, where $g(x)$ and $G(x)$ are the pdf and cdf of any continuous distribution to be generalized respectively and α and β are the two additional new parameters responsible for the shape of a distribution respectively.

Substituting equation (1) and (2) into equation (3) and (4) and simplifying, we obtain the cdf and pdf of the WEIED given in equation (5) and (6) respectively as:

$$F(x) = \exp\left\{ -\alpha \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta} \right\} \tag{5}$$

and

$$f(x) = \frac{\alpha\beta\theta\lambda e^{-\frac{\lambda}{x}} e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)}}{x^{2}\left[1-e^{-\frac{\lambda}{x}}\right]^{2}\left[1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)}\right]} \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta-1} \exp\left\{ -\alpha \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta} \right\} \tag{6}$$

where $x > 0, \alpha > 0, \theta > 0, \beta > 0$, α, θ and β are the shape parameters and λ is the scale parameter.

Plots of the pdf and cdf of the WEIED using some parameter values are presented in figure 1 as follows.

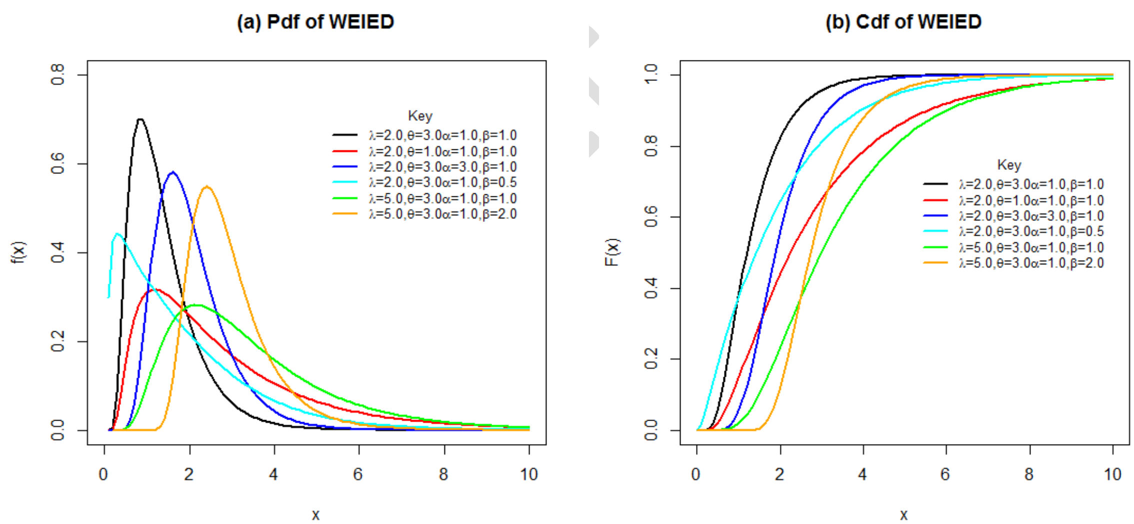

Fig. 1: (a) PDF and (b) CDF of the WEIED for different values of the parameters.

From the figure above, it is clear that the pdf of WEIED distribution is positively skewed and takes various shapes depending on the parameter values. Also, from the above plot of the cdf, it is seen that the cdf equals to one when *x* approaches infinity and equals zero when *x* tends to zero as normally expected.

3. Useful Statistical Properties of WEIED

In this section, useful properties of the WEIED distribution have been derived and discussed as follows:

3.1 Quantile Function

According to [25], the quantile function for any distribution is defined in the form $Q(u) = X_q = F^{-1}(u)$ where $Q(u)$ is the quantile function of $F(x)$ for $0 < u < 1$

Taking *F(x)* to be the cdf of the WEIED and inverting it as above will give us the quantile function as follows:

$$F(x) = \exp\left\{ -\alpha \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}} \right)} \right) \right)^{\beta} \right\} = u \qquad (7)$$

Simplifying equation (7) above and solving for x presents the quantile function of the WEIED as:

$$Q(u) = \lambda \left\{ \log\left[1 - \theta \left(\log\left(1 - \exp\left\{ -\left(-\frac{\log(u)}{\alpha} \right)^{\frac{1}{\beta}} \right\} \right) \right)^{-1} \right] \right\}^{-1} \qquad (8)$$

This function is used for obtaining some moments like skewness and kurtosis as well as the evaluation of median and for generation of random variables from the distribution.

3.2 Skewness and Kurtosis

This paper presents the quantile based measures of skewness and kurtosis due to non-existence of the classical measures in some cases.

According to [26], the Bowley's measure of skewness based on quartiles is given by:

$$SK = \frac{Q\left(\frac{3}{4}\right) - 2Q\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)} \qquad (9)$$

Also, the Moors kurtosis based on octiles proposed by [27] and is given by;

$$KT = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + \left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{1}{8}\right)} \qquad (10)$$

Where $Q(.)$ is obtainable with the help of equation (8).

3.3 Reliability analysis of the WEIED.

The Survival function describes the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \qquad (11)$$

Applying the cdf of the WEIED in (11), the survival function for the WEIED is obtained as:

$$S(x) = 1 - \exp\left\{ -\alpha \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta} \right\} \tag{12}$$

Hazard function is a function that describes the chances that a product or component will breakdown over an interval of time. It is mathematically defined as:

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} \tag{13}$$

Therefore, our definition of the hazard rate of the WEIED is given by

$$h(x) = \frac{\alpha\beta\theta\lambda e^{-\frac{\lambda}{x}} e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta-1} \exp\left\{ -\alpha \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta} \right\}}{x^2 \left[1 - e^{-\frac{\lambda}{x}} \right]^2 \left[1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right] \left\{ 1 - \exp\left\{ -\alpha \left(-\log\left(1 - e^{-\theta\left(\frac{e^{-\frac{\lambda}{x}}}{1-e^{-\frac{\lambda}{x}}}\right)} \right) \right)^{\beta} \right\} \right\}} \tag{14}$$

where $\lambda, \theta, \alpha, \beta > 0$.

The figure below presents a plot of both the survival function and hazard function of WEIED based on some selected parameter values as follows:

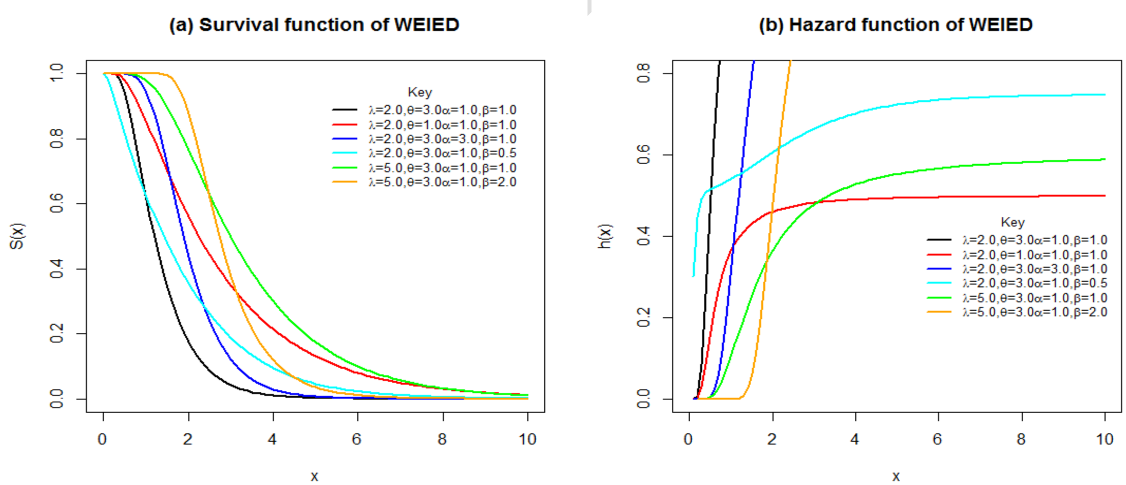

Figure 2: (a)-Survival function and (b)-Hazard function of WEIED for Selected Values of the Parameters.

The plot in figure **2(a)** show that the chances of survival equal are higher at the beginning or early age and decrease as the time increases and tends to zero at infinity. Figure 2(b) also revealed that the proposed distribution has increasing and constant failure rate which implies that the probability of failure for any random variable following a WEIED increases as time increases and could be constant for other parameter values.

4. Estimation of unknown Parameters of the WEIED

In this section, the estimation of the parameters of the WEIED is done by using the method of maximum likelihood estimation (MLE). Let $X_1, X_2, ..., X_n$ be a sample of size 'n' independently and identically distributed random variables from the WEIED with unknown parameters λ, θ, α and β defined previously.

The likelihood function of the WEIED using the pdf in equation (6) is given by;

$$L\left(\underline{X} / \alpha, \theta, \beta\right) = \frac{(\alpha\beta\theta\lambda)^n \prod_{i=1}^{n} \left\{ e^{-\frac{\lambda}{x_i}} e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)} \left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right)^{\beta-1} \exp\left\{ -\alpha \left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right)^{\beta} \right\} \right\}}{\prod_{i=1}^{n} \left(x_i^2 \left[1-e^{-\frac{\lambda}{x_i}}\right]^2 \left[1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right] \right)} \quad (15)$$

Let the natural logarithm of the likelihood function be, $l = \log L\left(\underline{X} \mid \lambda, \theta, \alpha, \beta\right)$, therefore, taking the natural logarithm of the function above gives:

$$l = n\log\lambda + n\log\theta + n\log\alpha + n\log\beta - 2\sum_{i=1}^{n}\log x_i - 2\sum_{i=1}^{n}\log\left(1-e^{-\frac{\lambda}{x_i}}\right) - \lambda\sum_{i=1}^{n} x^{-1}{}_i - \theta\sum_{i=1}^{n}\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)$$
$$+(\beta-1)\sum_{i=1}^{n}\log\left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right) - \alpha\sum_{i=1}^{n}\left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right) \quad (16)$$

Differentiating l partially with respect to λ, θ, α and β respectively gives the following results:

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x^{-1}{}_i + 2\sum_{i=1}^{n}\left\{\frac{x_i^{-1}e^{-\frac{\lambda}{x_i}}}{\left(1-e^{-\frac{\lambda}{x_i}}\right)}\right\} + \theta\sum_{i=1}^{n}\left\{\frac{x_i^{-1}e^{-\frac{\lambda}{x_i}}}{\left[1-e^{-\frac{\lambda}{x_i}}\right]^2}\right\} + \theta(\beta-1)\sum_{i=1}^{n}\left\{\frac{x_i^{-1}e^{-\frac{\lambda}{x_i}}\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)^{-1}}{\left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right)\left[1-e^{-\frac{\lambda}{x_i}}\right]^2}\right\} + \alpha\beta\theta\sum_{i=1}^{n}\left\{\frac{x_i^{-1}e^{-\frac{\lambda}{x_i}}\left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right)^{\beta-1}}{\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)e^{\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\left[1-e^{-\frac{\lambda}{x_i}}\right]^2}\right\} \quad (17)$$

$$\frac{\partial l}{\partial\theta} = \frac{n}{\theta} - \sum_{i=1}^{n}\left\{\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right\} - (\beta-1)\sum_{i=1}^{n}\left\{\frac{\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}}{\left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right)\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)}\right\} + \alpha\beta\sum_{i=1}^{n}\left\{\frac{\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)\left(-\log\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)\right)^{\beta-1}}{\left(1-e^{-\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}\right)e^{\theta\left(\frac{e^{-\frac{\lambda}{x_i}}}{1-e^{-\frac{\lambda}{x_i}}}\right)}}\right\} \quad (18)$$

$$\frac{\partial l}{\partial \alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\left(-\log\left(1-\mathrm{e}^{-\theta\left(\frac{\mathrm{e}^{-\frac{\lambda}{x_i}}}{1-\mathrm{e}^{-\frac{\lambda}{x_i}}}\right)}\right)\right)^{\beta} \tag{19}$$

$$\frac{\partial l}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\left(-\log\left(1-\mathrm{e}^{-\theta\left(\frac{\mathrm{e}^{-\frac{\lambda}{x_i}}}{1-\mathrm{e}^{-\frac{\lambda}{x_i}}}\right)}\right)\right)-\alpha\sum_{i=1}^{n}\left(-\log\left(1-\mathrm{e}^{-\theta\left(\frac{\mathrm{e}^{-\frac{\lambda}{x_i}}}{1-\mathrm{e}^{-\frac{\lambda}{x_i}}}\right)}\right)\right)^{\beta}\ln\left(-\log\left(1-\mathrm{e}^{-\theta\left(\frac{\mathrm{e}^{-\frac{\lambda}{x_i}}}{1-\mathrm{e}^{-\frac{\lambda}{x_i}}}\right)}\right)\right) \tag{20}$$

Equating (17), (18), (19) and (20) to zero (0) and solving for the solution of the non-linear system of equations above will give the maximum likelihood estimates $\hat{\lambda},\hat{\theta},\hat{\alpha}$ and $\hat{\beta}$ of parameters λ,θ,α and β respectively. However, these solutions cannot be obtained manually except numerically with the aid of suitable statistical software such as *R* software as used in this study.

5 Application to Mother-to-Child HIV Transmission Rate (MTCHIVTR)

This section presents a dataset on the rate of mother-to-child transmission of HIV (Human Immunodeficiency Virus) in Nigeria from the year 2000 to the year 2019. The descriptive statistics and graphs of the dataset are also presented.

The mother-to-child HIV transmission rate per 1,000 of population in Nigeria between 2000 and 2019 is as given below.

37.35, 37.08, 37.00, 36.98, 36.79, 36.75, 34.35, 32.96, 31.84, 30.35, 30.53, 28.96, 26.71, 22.50, 19.84, 20.04, 19.44, 20.82, 22.09, 22.16

Data source: www.data.unicef.org

The following table and figures present a good exploration of the dataset with some explanations.

Table 1: Summary statistics of the dataset

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Dataset A	20	19.44	22.14	30.44	36.76	29.23	37.35	47.55	-0.18919	-1.55278

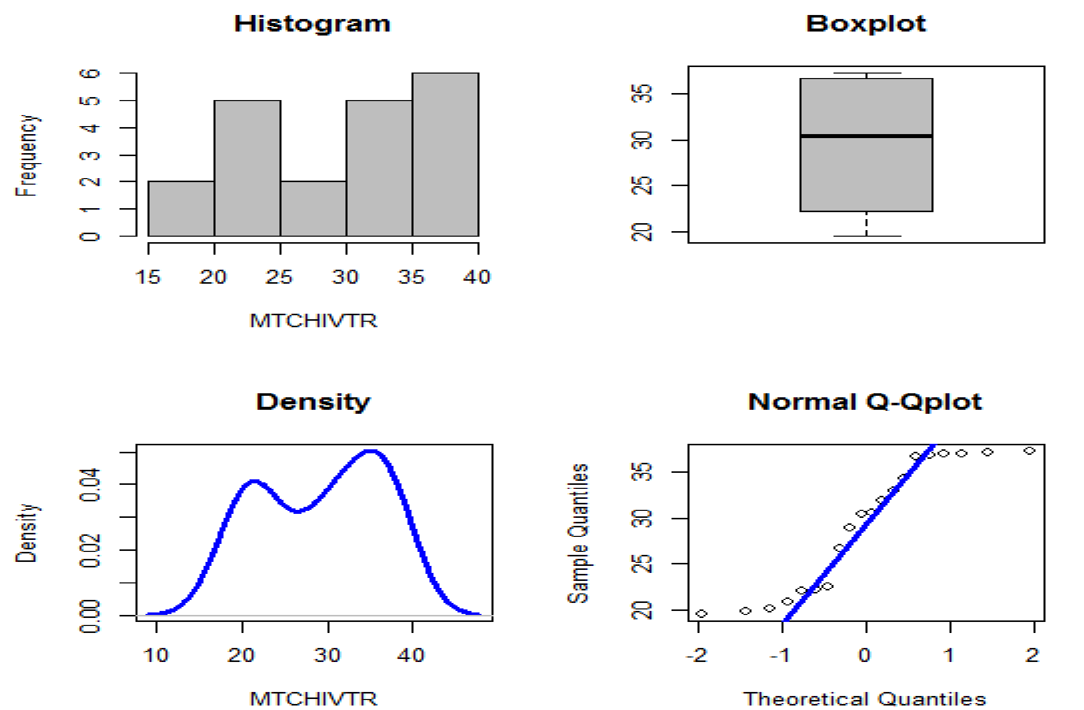

Figure 3: A graphical summary of the dataset

A summary of the dataset in table 1 and figure 3 has shown that the rate of transmission of HIV from mother to child is bimodal and approximately normally distributed.

Also, the trend in the rate of mother-to-child HIV transmission from 2000 to 2019 using a line plot is shown in figure 4 below.

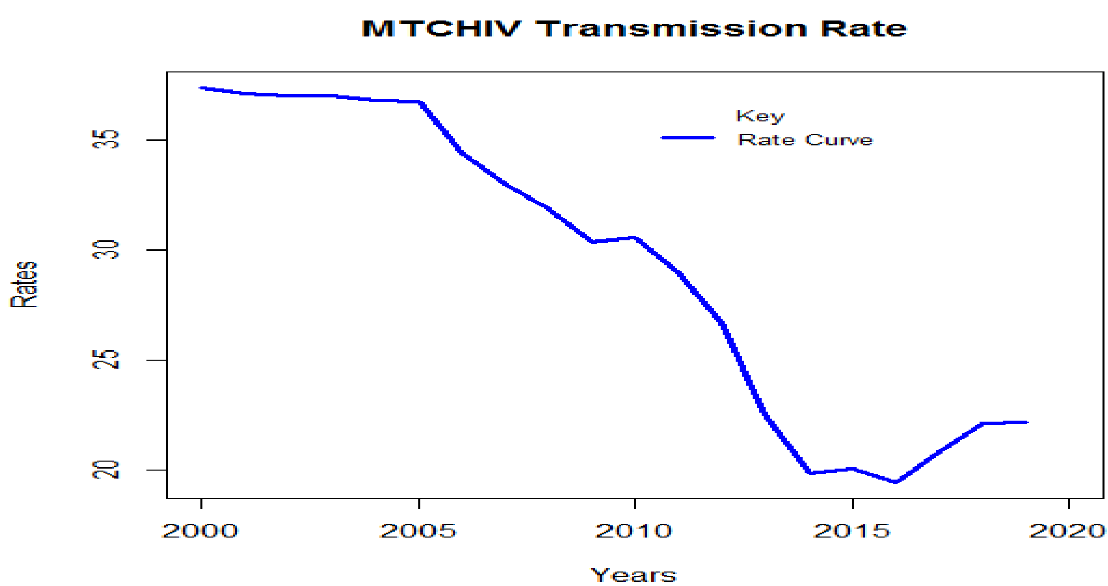

Figure 4: A Line plot of Mother-to-child HIV Transmission Rate in Nigeria from 2000 to 2019

From figure 4, the plot reveals the trend in the rate of mother-to-child transmission of HIV which shows that mother-to-child HIV transmission was a very big problem from the year 2000 to 2005 with a non-decreasing rate. Also, there came a decreasing trend in the rate of HIV transmission from mother to child as from the year 2006 to 2014, however, the trend from the year 2015 to 2019 is certainly an increasing move in the rate of mother-to-child transmission of HIV which predicts that more work have to be done by relevant stakeholders or organizations to drastically reduce or possibly eradicate the rate of mother-to-child HIV transmission in Nigeria.

Sequel to this challenge, this study fits the the proposed Weibull-Exponential Inverse Exponential distribution (WEIED) to the above dataset in comparison with other existing probability distributions such as Exponential Inverse Exponential distribution (EIED), Odd Lindley Inverse Exponential distribution (OLinED), Inverse Exponential distribution (IED), Weibull distribution (WD) and Exponential distribution (ED).

The model selection process has been done using the following model selection criteria: the value of the log-likelihood function evaluated at the MLEs (ℓ), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan Quin Information Criterion (HQIC), Anderson-Darling (A*), Cramèr-Von Mises (W*) and Kolmogorov-smirnov (K-S) statistics. Other details information on these criteria or statistics (A*, W* and K-S) can be found from [28].

Note: the probability model or distribution with the lowest values of these criteria is considered to be the best model that fit the dataset. Also, all the required computations are performed using the R package "AdequacyModel" which is freely available from http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf. The results from this R package and the commands are shown in tables as follows:
Tables 2 lists the Maximum Likelihood Estimates of the model parameters, table 3 presents the statistics AIC, CAIC, BIC and HQIC while A*, W* and K-S for the fitted models are given in Table 4.

Table 2: Maximum Likelihood Parameter Estimates From MTCHIVTR dataset

Distribution	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\beta}$
WEIED	9.2809134	0.5474425	9.3280471	2.0293554
OLinED	9.1879682	0.6327105	-	-
EIED	7.2246774	0.2803205	-	-
ExD	0.03421484	-	-	-
WD	-	-	0.0376618	0.7354662
IED	2.68577	-	-	-

Table 3: The statistics ℓ, AIC, CAIC, BIC and HQIC based on MTCHIVTR

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
WEIED	67.85642	143.7128	146.3795	147.6958	144.4904	1st
OLinED	81.13441	166.2688	166.9747	168.2603	166.6576	2nd
EIED	85.10735	174.2147	174.9206	176.2062	174.6035	3rd
ExD	87.50186	177.0037	177.2259	177.9994	177.1981	4th
WD	98.40054	200.8011	201.507	202.7926	201.1898	5th
IED	116.0583	234.1166	234.3388	235.1123	234.311	6th

Table 4: The A^*, W^*, K-S statistic and P-values based on the dataset used.

Distribution	A^*	W^*	K-S	P-Value (K-S)	Ranks
WEIED	1.1421	0.1795449	0.19121	0.4064	1st
OLinED	1.044315	0.1541706	0.42399	0.0008895	2nd
EIED	1.052061	0.1563206	0.46355	0.0001864	3rd
ExD	1.044722	0.1541682	0.4858	7.176e-05	4th
WD	1.058792	0.1578691	0.58284	5.526e-07	5th
IED	1.109782	0.1710509	0.87096	2.2e-16	6th

The following figure presents a histogram and estimated densities and cdfs of the fitted models to the MTCHIVTR dataset.

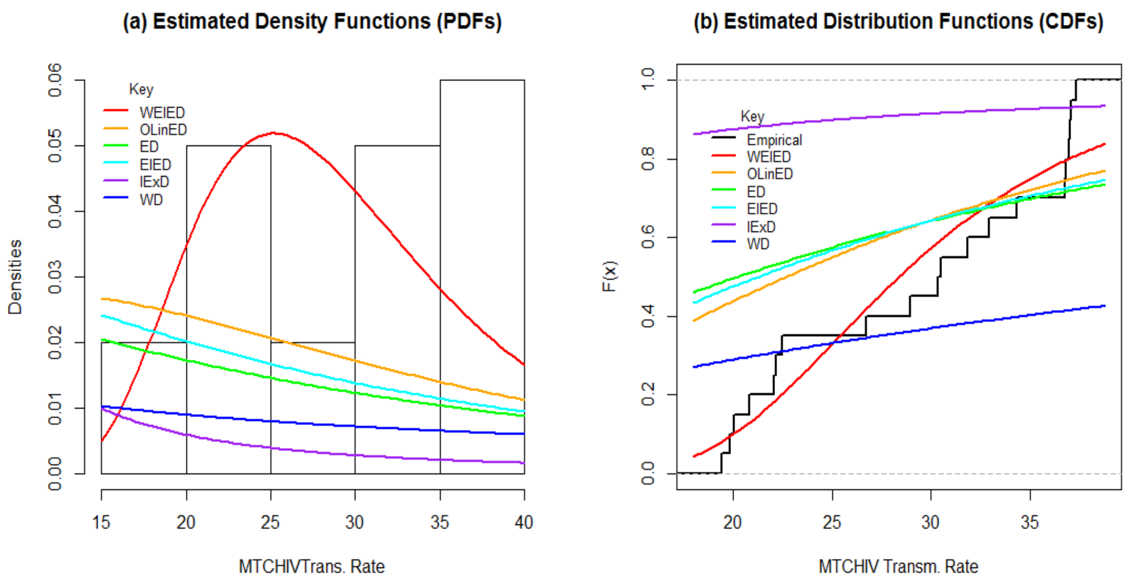

Figure 5: Histogram and plots of the estimated densities and cdfs of the fitted distributions to the dataset.

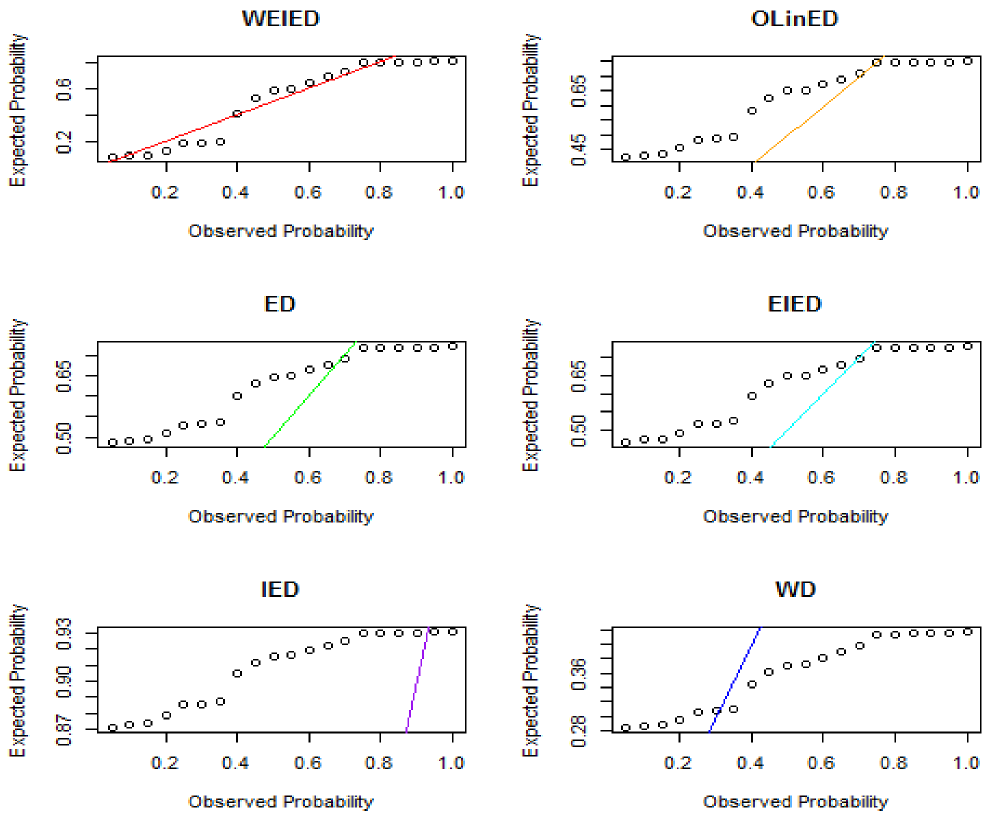

Figure 6: Probability plots for the six fitted distributions based on the MTCHIVTR dataset.

A brief discussion of the tables above is as follows: from the results in table 3 it is understood that the proposed distribution Weibull-Exponential inverse exponential distribution (WEIED) is better compared to the performance of the other fitted distributions following the values of the first four information criteria (AIC, CAIC, BIC and HQIC). Similarly, from the other model selection criteria in Tables 4, it is also clear that the WEIED has the minimum values of A^*, W^* and K-S statistic compared to every other model fitted to the MTCHIVTR dataset. Based on these model selection criteria, it is clear that the WEIED has the overall best fit to the mother-to-child HIV transmission rate dataset and therefore it is taken as the most appropriate model for analyzing this MTCHIVTR dataset as considered in this study.

Also, the histogram of the MTCHIVTR dataset together with the fitted densities and estimated cumulative distribution functions given in figure 5 prove that the proposed model analyses the dataset better than the other five distributions. Furthermore, the probability plots in figure 6 are evidences that the proposed distribution (WEIED) is more flexible than the other five distributions (OLinED, EIED, ED, WD and IED) as already revealed previously by the statistics in Tables 3 and 4 as well as in figure 5.

From our results in this study, it is true that adding parameter(s) to most continuous probability distributions leads to more flexible distributions that could be used in modeling real life data which is in line with many other previous results.

7 Conclusion

Just as discussed above, this paper developed a new distribution called "a Weibull-Exponential Inverse Exponential distribution". The statistical properties of this model which are useful have been derived and studied. The quantile function, coefficient of skewness and kurtosis, survival function and hazard function were defined and discussed in this paper. The unknown parameters of the proposed model were estimated using the method of maximum likelihood estimation. The WEIED was used to fit a dataset on mother-to-child HIV transmission rate in Nigeria from the year 2000 to 2019 in comparison with other existing extensions of the inverse exponential distribution. It was also discovered that the rate of mother-to-child transmission of HIV in Nigeria is on the increase following its trend from 2000 to 2019 and there is need for immediate actions from relevant health agencies or organizations. Our results from application of the proposed model to the HIV transmission data reveal that the Weibull-exponential inverse exponential distribution fits the dataset much better than the other five fitted distributions. This fitting ability of our model is an indication that the proposed model will be useful for describing other medical situations and survival analysis in general.

REFERENCES

[1] Cordeiro GM, Afify AZ, Ortega EMM, Suzuki AK & Mead ME. The odd Lomax generator of distributions: Properties, estimation and applications. Journal of Computational and Applied Mathematics, 2019; 347: 222–237.

[2] Cordeiro GM, Ortega EMM, Popovic BV & Pescim RR. The Lomax generator of distributions: Properties, minification process and regression model. *Applied Mathematics and Computation*, 2014; 247: 465-486.

[3] Abdullahi UK & Ieren TG. On the inferences and applications of transmuted exponential Lomax distribution, *International Journal of Advanced Probability and Statistics*, 2018; 6(1): 30-36.

[4] Yahaya A & Ieren TG. On Transmuted Weibull-Exponential Distribution: Its Properties and Applications, *Nigerian Journal of Scientific Research*, 2017; 16(3): 289-*297*.

[5] Afify MZ, Yousof HM, Cordeiro GM, Ortega EMM. & Nofal ZM. The Weibull Frechet Distribution and Its Applications. Journal of Applied Statistics, 2016; 1-22.

[6] Ieren TG & Kuhe AD. On the Properties and Applications of Lomax-Exponential Distribution. *Asian Journal of Probability and Statistics*, 2018; 1(4): 1-13.

[7] Tahir MH, Zubair M, Mansoor M, Cordeiro GM & Alizadeh M. A New Weibull-G family of distributions. *Hacettepe Journal of Mathematics and Statistics*, 2016; 45(2), 629-647.

[8] Ieren TG & Yahaya A. The Weimal Distribution: its properties and applications. *Journal of the Nigeria Association of Mathematical Physics*, 2017; 39: 135-148.

[9] Bourguignon M, Silva RB & Cordeiro G. M. The weibull-G family of probability distributions. Journal of Data Science, 2014; 12: 53-68.

[10] Gomes-Silva F, Percontini A, De Brito E, Ramos MW, Venancio R & Cordeiro GM. The Odd Lindley-G Family of Distributions. Austrian Journal of Statistics, 2017; 46, 65-87.

[11] Ieren TG, Koleoso PO, Chama AF, Eraikhuemen IB & Yakubu N. A Lomax-inverse Lindley Distribution: Model, Properties and Applications to Lifetime Data. *Journal of Advances in Mathematics and Computer Science*, 2019; 34(3-4): 1-28.

[12] Alzaatreh A, Famoye F & Lee C. A new method for generating families of continuous distributions. *Metron*, 2013; 71: 63–79.

[13] Ieren TG, Kromtit FM, Agbor BU, Eraikhuemen IB & Koleoso PO. A Power Gompertz Distribution: Model, Properties and Application to Bladder Cancer Data. *Asian Research Journal of Mathematics*, 2019; 15(2): 1-14.

[14] Ieren TG, Oyamakin SO & Chukwu AU. Modeling Lifetime Data With Weibull-Lindley Distribution. *Biometrics and Biostatistics International Journal*, 2018; 7(6): 532–544.

[15] Oguntunde PE, Adejumo AO, & Owoloko EA. Exponential Inverse Exponential (EIE) distribution with applications to lifetime data, *Asian Journal Scientific Research*, 2017; 10: 169-177.

[16] Ieren, T. G. & Abdullahi, J. Properties and Applications of a Two-Parameter Inverse Exponential Distribution with a Decreasing Failure Rate. Pakistan Journal of Statistics, 2020; 36(3): 183-206.

[17] Oguntunde, P. E., Adejumo, A. O. and Owoloko, E. A. Application of Kumaraswamy inverse exponential distribution to real lifetime data. *International Journal of Applied Mathematics and Statistics*, 2017; 56(5), 34–47

[18] Oguntunde, P. E., Adejumo, A. O. and Owoloko, E. A. On the exponentiated generalized inverse exponential distribution-Lecture Notes on engineering and computer science. *Proceeding of the World Congress on Engineering* 2017; *(pp. 80–83).* London, UK.

[19] Oguntunde, P. E., Adejumo, A. O., Okagbue, H. and IRastogi, M. K. Statistical Properties and Applications of a New Lindley Exponential Distribution. *Gazi University Journal of Science, 2016; 29(4), 831-838.*

[20] Abdullahi, J., Abdullahi, U. K., Ieren, T. G., Kuhe, D. A. and Umar, A. A. On the properties and applications of transmuted odd generalized exponential- exponential distribution. *Asian Journal of Probability and Statistics*, 2018; 1(4):1-14.

[21] Owoloko, E. A., Oguntunde, P. E. and Adejumo, A. O. Performance rating of the transmuted exponential distribution: an analytical approach. *Spring*erplus 2015; 4, 818-829.

[22] Oguntunde, P. E. and Adejumo, A. O, The transmuted inverse exponential distribution. *International Journal of Advanced Statistics and Probability*, 2015; 3(1), 1–7.

[23] Maiti, S. S. and Pramanik, S. Odds Generalized Exponential-Exponential Distribution. *Journal of Data Science*, 2015; 13, 733-754.

[24] Oguntunde, P. E., Balogun, O. S., Okagbue, H. I. and Bishop, S. A. The Weibull-Exponential Distribution: Its properties and application. *Journal of Applied Sciences*, 2015; 15(11), 1305-1311.

[25] Hyndman. R.J., and Fan, Y. Sample quantiles in statistical packages. *The American Statistician*, 1996; 50(4), 361-365.

[26] Kenney, J. F., and Keeping, E. S. Mathematics of Statistics. 3 edn, *Chapman & Hall Ltd, New Jersey*, 1962.

[27] Moors, J. J. A quantile alternative for kurtosis. *Journal of the Royal Statistical Society*: 1988; *Series D*, 37, 25–32.

[28] Chen, G., Balakrishnan, N. A general purpose approximate goodness-of-fit test. *Journal of Quality Technology*, 1995; 27, 154–161