Original Research Article

Forecasting of the Nigeria Stock Returns Volatility Using GARCH Models with Structural Breaks

ABSTRACT

This study examines the stock returns series using Symmetric and Asymmetric GARCH models with structural breaks in the presence of some varying distribution assumptions. Volatility models of Symmetric GARCH (1,1), Asymmetric Power GARCH (1,1) and GJR-GARCH(1,1) models were considered in estimating and measuring shock persistence, leverage effects and mean reversion rate with structural breaks considering dummy variable for these structural changes and varying distributions . The skewed student-t distribution is considered best distribution for the models; moreover findings showed the high persistence of shock in returns series for the estimated models. However, when structural breaks were incorporated in the estimated models by including dummy variable in the conditional variance equations of all the models, there was significant reduction of shock persistence parameter and mean reversion rate. The study found the GJR-GARCH (1,1) with skewed student-t distribution best fit the series. The volatility was forecasted for 12 months period using GJR-GARCH (1,1) model and the values are compared with the actual values and the results indicates a continuous increase in unconditional variance.

Keywords: Volatility, *GARCH, Structural Breaks, Nigeria*

1.0 INTRODUCTION

Volatility is a key indicator in assessing the performance of the stock market in order for both indigenous and foreign speculators to make accurate speculations and decisions on investments [1]. Over the past years, modeling and forecasting volatility of a financial time series has become a popular area of research and has gained a great deal of attention , this is because volatility is considered as an important concept for many economic and financial applications, like risk management, portfolio optimization and asset pricing. The issue with volatility of stock market returns refers to the fluctuations that may be observed in stock market returns over time. The major reason for the ups and downs in the stock market may be traced to macroeconomic instability. Since the stock market operate in a macroeconomic environment, it is therefore necessary that the environment must be an enabling one in order to realize its full potentials.

Engle in 1982, introduced Autoregressive Conditional Heteroscedasticity (ARCH) model to the world to model financial time series that exhibit time varying conditional variance [2]. A generalized ARCH (GARCH) model extended by [3] is another popular model for estimating stochastic volatility. These models are generally used in various branches of econometrics, especially in financial time series analysis. Besides, with the introduction of models of ARCH and GARCH, there have been number of empirical applications of modeling variance (volatility) of financial time series. Though, the GARCH cannot account for leverage effect, however they account for volatility clustering and leptokurtosis in a series, this necessitated to the development of new and extended models over GARCH that resulted in to new models such as: GJR- GARCH, PGARCH, EGARCH and many others.

Empirically, some reviewed literature show that symmetric and asymmetric GARCH models have been applied to financial, fiscal and economic variables. The study by [4], used Independent Component Analysis (ICA) to transformed six items in the Composite Consumer Price Index of Nigeria in to statistically independent time series and six different univariate GARCH models were fitted to the Independent Components. Their finding showed that all the returns series have excess kurtosis and presence of ARCH effects and the best fit model for all Independent Component are found to be EGARCH model with student-t distribution.

In [5] , they evaluated the adequacy of volatility models of Nigeria Stock Market using Principal Component Analysis and Ranking Method to produce a new set of variables called principle components formed by linear combinations of the statistical criterion which ranked the values and choose the most appropriate models that fit the time series. Their finding revealed that the best fitted model and the worse fitted model for the training period were CGARCH (1,1) and ARCH (1) while the result for the testing period indicate ARCH (1) and GARCH (2,1) were the most appropriate. [6], examined the impact of structural breaks on the conditional variance of daily stock returns of 8 commercial banks in Nigerian stock market. They employed symmetric GARCH, asymmetric EGARCH and TGARCH models with and without dummy variables to evaluate variance persistence, mean reversion, asymmetry and leverage effects. Results showed high persistence in conditional volatility for the banking stocks, but when the random level shifts were incorporated into the models, there was reduction in the conditional volatility of these models. . In [7], they investigated the performances of different GARCH models while estimating the volatility of headline and core CPI inflation in Nigeria for the period 1995M01 to 2016M10 using ADF breakpoint testing procedure. The study applied both symmetric and asymmetric GARCH variants, and observed empirical evidence of shock persistence in both CPI stock returns with the presence of leverages only in the headline CPI return series.

Examination of the monthly volatility of Naira/Dollar exchange rates in Nigeria between the periods of January, 1995 to December, 2016 was carried out by [8]. The traditional GARCH and dynamic neural networks were hybridized to developed the propose model for forecasting volatility of inflation rates in Nigeria. The study applied both symmetric and asymmetric GARCH, the value of the volatility estimated by the best fitted GARCH as an input to the neural network. The forecasts obtained by each of those hybrid models have been compared with those of GARCH models in terms of the actual volatility. The computational result demonstrates that the second hybrid model provides better volatility forecasts. In 2016 [9], shock persistence and asymmetry in Nigerian stock market by incorporating structural breaks was investigated, using monthly stock returns for the period from January 1985 to December 2014. Result from the basic GARCH model showed higher shock persistence during pre-break sub-periods, than the post-break sub-periods. No evidence of asymmetry or leverage effect was found in the asymmetric GARCH model with or without incorporating the breakpoints in Nigerian stock market.

The study by [10], modeled abrupt shift in time series using dummy variable by employing both symmetric and asymmetric GARCH models with and without sudden shifts in variance. They used daily quotations of 10 insurance stocks of the Nigerian stock exchange. The study found significant reduction in shock persistence in volatility of most insurance stock returns when the regime shifts were incorporated into the models. Adesina in 2013 used symmetric and asymmetric GARCH models to estimate the stock return volatility and the persistence of shocks to volatility of the Nigerian Stock Exchange (NSE). There is substantial evidence for the GARCH modeling through Lagrange Multiplier Test, Correlogram and Ljung-Box Statistics before the estimation of the GARCH models. The study uses 324 monthly data from January 1985 to December 2011 of the NSE all share-index. The result reveals high persistent volatility for the NSE return series. In addition, there is no asymmetric shock phenomenon (leverage effect) for the return series [11].

The aim of this research is to use the Symmetric and Asymmetric GARCH models with structural breaks to model the volatility of the Nigeria stock market returns presence of varying distribution assumptions.

2.0 MATERIALS AND METHODS

The Multiple Breakpoints Test, symmetric and asymmetric GARCH-type models are briefly discussed following [6].

2.1 Bai and Perron Multiple Breakpoints Test

Bai and Perron in 1998, developed a multiple structural breakpoints testing procedure, which predict persistently several shifts in variance [12]. The power of the test was strengthened by [13], which made the test more efficient. The model considered is the multiple linear regression models with m breaks or m + 1 regimes.

$$y = x_i^T \beta_i + u_t \quad (1)$$

$$y_i = x_i^T \beta + Z_i^T \delta + u_t \quad (2)$$

Where $u_t \sim iid(0, \sigma^2)$is the response variable at time i = 1,2,3, . . . ,n and x_i= $[1, x_{i2}, x_{i3}, x_{i4} \; . \; . \; . \; x_{ik}]^T$ is a vector of order k ×1 of independent variables having one as its initial value and β_i is also k ×1 vector of coefficients. The hypothesis for random level shift is: H_0

:$\beta_i = \beta_0$ for i =1,2,3, . . . ,n (i.e., there is no structural break in the series) versus alternative that with the random level shift in time the vector of coefficients also changes, also assuming that they have no stochastic behavior as a departure from the null hypothesis. i.e.

$$\|x_i\| = O(1)\ and\ that\ \frac{1}{n}\sum_{i=1}^{n} x_i x_i^T \to Z \qquad (3)$$

Where Z represents a finite matrix. This expression permits the detection of multiple breakpoints in data and once the breakpoints are recognized, they will be incorporated into each GARCH model in order to avoid spurious results.

2.2 GARCH (p,q) Model

GARCH models are useful in capturing the leptokurtic nature of financial time series data as well as volatility clustering and help in modeling the changing conditional variances in time series [3]. The general GARCH (p,q) model is defined as, assuming log returns series $r_t = \mu + \varepsilon_t$ where ε_t is the error term at time t.

$$\sigma_t^2 = \Phi_0 + \sum_{i=1}^{p} \Phi_i u_{t-i}^2 + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2 \qquad (4)$$

Where

$$\Phi_0 > 0,\ \Phi_1 and\ \ \theta_j \geq 0 \quad , \quad \sum_{i=1}^{\max(p,q)} (\Phi_i + \theta_i) < 1$$

The GARCH(p,q) “p” is the order of the past residual term while the “q” the order of the past conditional variance. The GARCH model allows the error variance (σ_t^2) depending on either its own past squared errors (u_{t-p}^2) or its own past values (σ_{t-q}^2) . GARCH also assumes that the variance is non-negative, large q order signs that shocks to the conditional variance take a long time to die out meaning highly persistent volatility, while large p order implies a sizeable reaction of volatility to market movement. Hence, if $\boldsymbol{\Phi}_{\mathrm{i}}$+ $\boldsymbol{\theta_j}$ is close to unity, the shock at t time will be persistent for many future periods.

The GARCH (p,q) model with dummy variable in the conditional variance is specified as:

$$\sigma_t^2 = \Phi_0 + \alpha_1 N_1 + \ldots + \alpha_n N_n + \sum_{i=1}^{p} \Phi_i u_{t-i}^2 + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2$$

where $N_1 \ldots N_n$ are dummy variables added to the conditional varianceequation which takes value 1 as the sudden break appears in conditionalvolatility onwards and otherwise it takes value 0.

To remedy some weakness of symmetric GARCH model, others asymmetric GARCH model are:

(a) **Exponential GARCH (EGARCH) (p,q) : [14]** advanced the model as:

$$\boldsymbol{ln}(\boldsymbol{\sigma_t^2}) = \Phi_0 + \sum_{i=1}^{p} \Phi_i \frac{|u_{t-i}| + \gamma_i u_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \theta_j\, ln(\boldsymbol{\sigma_{t-j}^2}) \qquad (5)$$

The presence of parameter γ_i indicates an asymmetric effect of shocks on volatility.

The EGARCH(p,q) with dummy variable in the conditional variance is specified as

$$\boldsymbol{ln}(\boldsymbol{\sigma_t^2}) = \Phi_0 + \alpha_1 N_1 + \ldots + \alpha_n N_n + \sum_{i=1}^{p} \Phi_i \frac{|u_{t-i}| + \gamma_i u_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \theta_j\, ln(\boldsymbol{\sigma_{i-j}^2})$$

where $N_1 \ldots N_n$ are dummy variables added to the conditional variance equation which takes value 1 as the sudden break appears in conditional volatility onwards and otherwise it takes value 0.

(b) GJR-GARCH (p,q) Model: Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model by [15]. The general GJR-GARCH (p,q) model assumes that the conditional variance at time t follows:

$$\sigma_t^2 = \Phi_0 + \sum_{i=1}^{p} \Phi_i u_{t-i}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} u_{t-i}^2 + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2 \qquad (6)$$

$I_{t-j} = 1\ if u_{t-i}^2 < 0, I_{t-i} = 0 \quad and \quad \Phi_0 > 0, \gamma_i \geq 0, \theta_j \geq 0,\ \theta_j + \gamma_I \geq 0, \mathrm{i} = 1, \ldots, \mathrm{p}; \mathrm{j} = 1, \ldots, \mathrm{q}. \mathrm{so}\ \gamma_j > 0.$

Note that GARCH and GJR-GARCH models allow for volatility clustering (i.e. persistence) by a combination of the Φ_I and θ_J terms.

The GJR-GARCH (p,q) with dummy variable in the conditional variance is specified as

$$\sigma_t^2 = \Phi_0 + \alpha_1 N_1 + ... + \alpha_n N_n + \sum_{i=1}^{p} \Phi_i u_{t-i}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} u_{t-i}^2 + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^2$$

Where$N_1 .\ .\ .\ .\ N_n$dummy variables are added to the conditional variance equation which takes value 1 as the sudden break appears in conditional volatility onwards and otherwise it takes value 0.

(c) Power GARCH (PGARCH (p,d.q)): Power GARCH (PGARCH) Model is another class of ARCH extensive model which is capable of forecasting volatility index.

$$\sigma_t^d = \Phi_0 + \sum_{i=1}^{p} \Phi_i (|u_{t-i}| + \gamma_i u_{t-i})^d + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^d \qquad (7)$$

If d = 2, then the PGARCH mimics a GARCH (p, q) with a leverage effect .
The Power GARCH(p,q) with dummy variable in the conditional variance is specified as

$$\sigma_t^d = \Phi_0 + \alpha_1 N_1 + ... + \alpha_n N_n + \sum_{i=1}^{p} \Phi_i (|u_{t-i}| + \gamma_i)^d + \sum_{j=1}^{q} \theta_j \sigma_{t-j}^d$$

Where $N_1, .\ .\ ., N_n$ dummy variables are added to the conditional variance equation which takes value 1 as the sudden break appears in conditional volatility onwards and otherwise it takes value 0.

2.3 Forecast Error Statistic

The forecast error statistic used is Mean Square Error (MSE) defined as

$$MSE = \frac{1}{M} \sum_{t=1}^{m} (\hat{y}_t - y)^2 \qquad (8)$$

The MSE depend on the scale of dependent variable and differences between volatility value and forecasted value. The smaller the error statistic is, the better the forecasting ability of the model in consideration of that measure.

3.0 DATA ANALYSIS AND DISCUSSION

The data employed for this work are the monthly All Share Index (ASI) of Nigeria Stock Exchange (NSE) from January, 1985 to April, 2019, resulting in 412 observations. The monthly ASI series are used to generate the continuously compounded returns as follows:

$$R_t = 100 * \log\left(\frac{\mathrm{p_t}}{\mathrm{p_{t-1}}}\right) = 100[\log(\mathrm{p_t}) - \log(\mathrm{p_{t-1}})] \qquad (9)$$

Monthly return of ASI for the period t and $p_t\ and\ p_{t-1}$ represent ASI for current month and previous month respectively.

Table 1: Structural Breaks in Volatility with Time period.

Returns	Break Points	Time Period
Monthly Returns	9	1. August, 1987 2. August, 1993 3. August, 2008 4. March, 2009 5. November, 2015 6. November, 2017 7. March, 2018 8. June, 2018 9. February, 2019

The multiple breakpoints test to the returns series detected a maximum of 9 break point and their dates for the returns series.

Table 2: Result of Structural Break Test

Test Statistic		P-Value
F-Statistic	4.947673*	0.0000
Loglikelihood Ratio	43.27873*	0.0000
Wald Test	44.52905*	0.0000

The signs * denote rejection of null hypothesis at 1 and 5 percent level of significance since the p-value is less than the critical value and concluded that there is structural breaks at the above specified break points. The reasons for these sudden structural breaks are the crude oil price fluctuations in the country, the global financial crises also affected the Nigerian stock market, economic recession, insecurity problem, and other reasons were as a result of internal, local, domestic, political or economic crises in the country.

3.1 Model Selection

The selections of best fitting symmetric and asymmetric GARCH models with suitable distributional assumption: Student -t Distribution (STD), Skew Student -t Distribution (SSTD), and Generalized Error Distribution (GED) were made using information criteria such as Akaike information criterion (AIC) , Bayesian information criterion (BIC) and log likelihoods (LogL). The selected models were presented in the following table.

Table 3: Model Selection

S/N	Model	Distributions	LogL	AIC	BIC
1	**GARCH(1,1)***	**Skewed Student-t Distribution**	**-263.563**	**12.175**	**12.175**
2	**PGARCH (1,1)***	**Skewed Student-t Distribution**	**-201.124**	**10.160**	**10.161**
3	**GJR-GARCH (1,1)***	**Skewed Student-t Distribution**	**-996.224**	**10.125**	**10.125**

3.2 Symmetric and Asymmetric Volatility Estimate with Structural Breaks

The detected structural breaks are considered in the volatility models by incorporating indicator (dummy) variable in the conditional variance equations of the symmetric GARCH (1,1), PGARCH (1,1) and GJR-GARCH (1,1) models.

Table 4: Parameter Estimate of Symmetric GARCH models with Structural Breaks

Parameter	**Coefficient**	**Std Error**	**Z- Statistic**	**P-value**
GARCH (1,1) model with Skewed Student-t Distribution				
Mean Equation				
μ	-0.0024	0.0010	-0.0126	0.00106
Variance Equation				
ϕ_0	0.0167	0.0026	4.1634	0.0000
A	-0.0413	0.0038	3.1262	0.0000
ϕ_1	0.0129	0.0312	4.5730	0.0000
θ_1	0.3162	0.0559	3.6440	0.0000
V	3.1652	0.3125	4.5920	0.0000
$\phi_1 + \theta_1$	0.3291			
ARCH LM Test			0.05989	0.8027

From Table 4, it was observed that there are significant decreases in the values of shock persistence parameters (θ) and mean reversion rates ($\phi_1 + \theta_1$) in all estimated parameter of symmetric GARCH models of the stock market returns. When included the structural breaks in these models, the stationarity and stability conditions of the models are satisfied as the sum of ARCH and GARCH terms were less than one in all the estimated models with breaks. This shows that the conditional variance process was stable and predictable and that the memories of volatility shocks were remembered in Nigerian stock market. Mean reverting and stationary stock returns were good for long term investment.

Table 5**:** Parameter Estimate of PGARCH models with Structural Breaks

Parameter	**Coefficient**	**Std Error**	**Z- Statistic**	**P-value**
Power GARCH (1,1) model with Skewed Student-t Distribution				
Mean Equation				
μ	-0.0421.	0.0021	-0.0042	0.0050
Variance Equation				
ϕ_0	-0.2102	0.0016	-12.1634	0.0000
A	-0.00211	0.00342	8.13124	
ϕ_1	0.1045	0.0012	4.5812	0.0000
γ	0.0063	0.0216	11.500	0.0012
θ_1	0.4612	0.0152	3.644	0.0000
V	4.3462	0.3128	22.5863	0.0000
$\phi_1 + \theta_1$	0.5657			
ARCH LM Test			0.01521	0.8635

The coefficients of the dummy variable (α) was negative and statistically significant in estimated PGARCH (1,1) model impaired the stock return series had negatively affected the Nigerian stock market during the study period. The stock return series retained the fat-tailed behavior even after incorporating the sudden shifts in variance as the shape parameter v = 4.3462 > 2 for PGARCH model, this clearly indicated that the Nigerian stock returns were heavy-tailed, as one of the stylized facts of financial returns common in developed markets.

Table 6**:** Parameter Estimate of GJR-GARCH models with Structural Breaks

Parameter	**Coefficient**	**Std Error**	**Z- Statistic**	**P-value**
GJR GARCH (1,1) model with Skewed Student-t Distribution				
Mean Equation				
μ	-0.0604	0.0062	-2.2014	0.0015
Variance Equation				
ϕ_0	0.0162	0.0056	2.1634	0.0000
A	-0.0103	0.0413	6.1201	0.0000
ϕ_1	-0.0284	0.0312	-15.588	0.0000
γ	0.1563	0.0286	7.500	0.0012
θ_1	0.3109	0.0559	-5.6265	0.0000
V	-0.0363	0.3125	12.5283	0.0000
$\theta_1+\phi_1+\gamma/2$	0.36075			
ARCH LM Test			0.05989	0.8027

The coefficients of the dummy variable (α) was negative and statistically significant in model suggesting the necessary factor mentioned that causes unexpected changes impairing the stock return series had negatively affected the Nigerian stock market during the study period. The stock return series retained the fat-tailed behavior even after incorporating the sudden shifts in variance as the shape parameter $v = 3.1265 > 2$ for GJR-GARCH model. This clearly indicates that the Nigerian stock returns were heavy-tailed, one of the stylized facts of financial returns common in developed markets.

3.3 Volatility forecast

The actual value of volatility and forecasted values of volatility Using GJR-GARCH model with all the Distributions are presented in the following tables and compared. The accuracy of the forecasting technique is measured using the Mean Square Error (MSE) parameter. The model distribution with the lowest MSE value for all 12 forecasted values is considered as accurate forecasting model.

Table 7: Actual and Forecasted volatility values for 12 months

Monthly Forecast	Actual Volatility	Student-t Distribution	Skew Student-t Distribution	Generalized Error Distribution
T+1	-1.0037	0.9289	0.9289	0.9289
T+2	-1.2346	1.0449	1.0449	1.0449
T+3	-0.2481	1.0681	1.0681	1.0681
T+4	-3.4636	1.0728	1.0728	1.0728
T+5	0.1978	1.0737	1.0737	1.0737
T+6	0.1278	1.0739	-1.0739	1.0739
T+7	-2.6226	1.0739	-1.0739	-2.0729
T+8	-2.6755	1.0739	-1.0739	-2.0729
T+9	-0.3995	1.0739	1.0739	-2.0729
T+10	-2.1837	1.0739	1.0739	1.0739
T+11	0.7755	1.0739	1.0739	1.0739
T+12	0.1379	1.0739	1.0739	0.0739

Table 8: Mean Square Error (MSE) of the Forecasted values

Error Rate	**Student-t Distribution**	**Skew Student-t Distribution**	**Generalized Error Distribution**
T+1	0.000233	0.000000	0.044422
T+2	0.001507	0.000410	0.216505
T+3	0.072182	0.000362	0.072182
T+4	0.080596	0.046051	0.857455
T+5	0.031966	0.000416	0.031966
T+6	0.037296	0.001666	0.037296
T+7	0.919046	0.010416	0.569338
T+8	0.939866	0.000416	0.585750
T+9	0.090454	0.000153	0.090454
T+10	0.755298	0.000000	0.442164
T+11	0.003710	0.000000	0.003710
T+12	0.036504	0.000000	0.036504

The GJR-GARCH model with skew student-t distribution having lowest MSE is considered as the accurate model compared to other models.

Table 9**:** Feature Volatility Forecast Using GJR-GARCH

NO. Forecast	**Feature Series Forecast**	**Sigma Forecast**
T+1	-0.78401	2.736
T+2	-0.25495	2.847
T+3	0.08014	2.953
T+4	0.29237	3.056
T+5	0.42680	3.551
T+6	0.51193	3.344
T+7	0.56586	3.435
T+8	0.60001	3.523
T+9	0.62104	3.609
T+10	0.63534	3.693
T+11	0.64402	3.721
T+12	0.64952	3.775

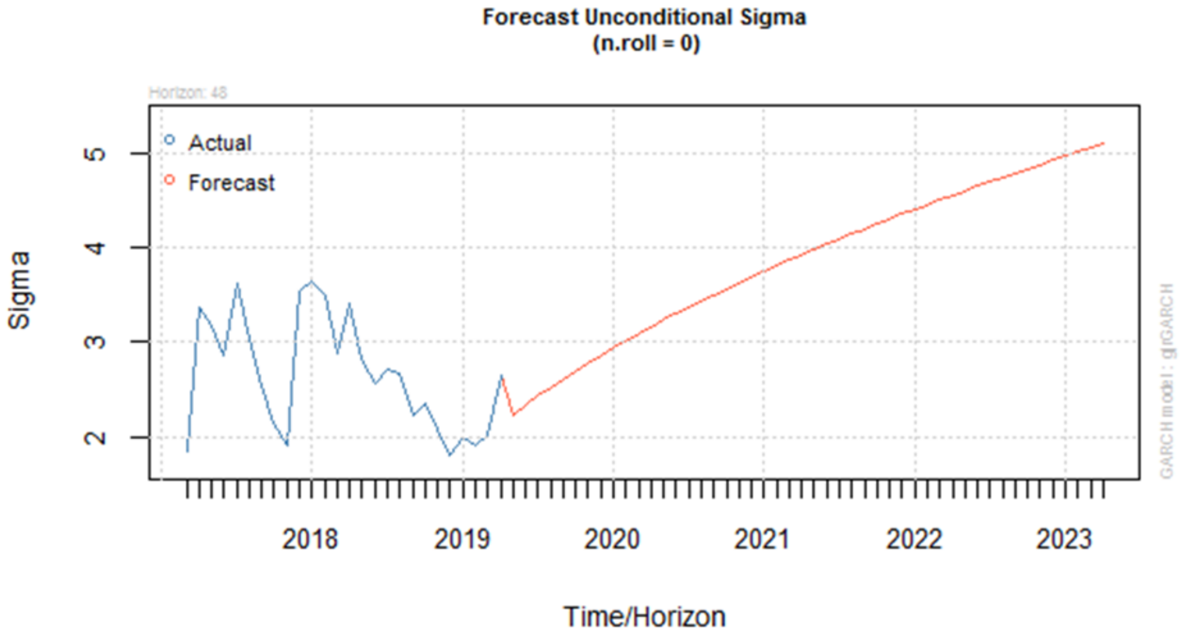

Figure 1: Forecast Plot

When we carefully look at the figure 1 the blue line represents the plot of actual returns series and the red line indicates the square root of unconditional variance and we can have observed from the forecast plot there is continuous increase in unconditional variance.

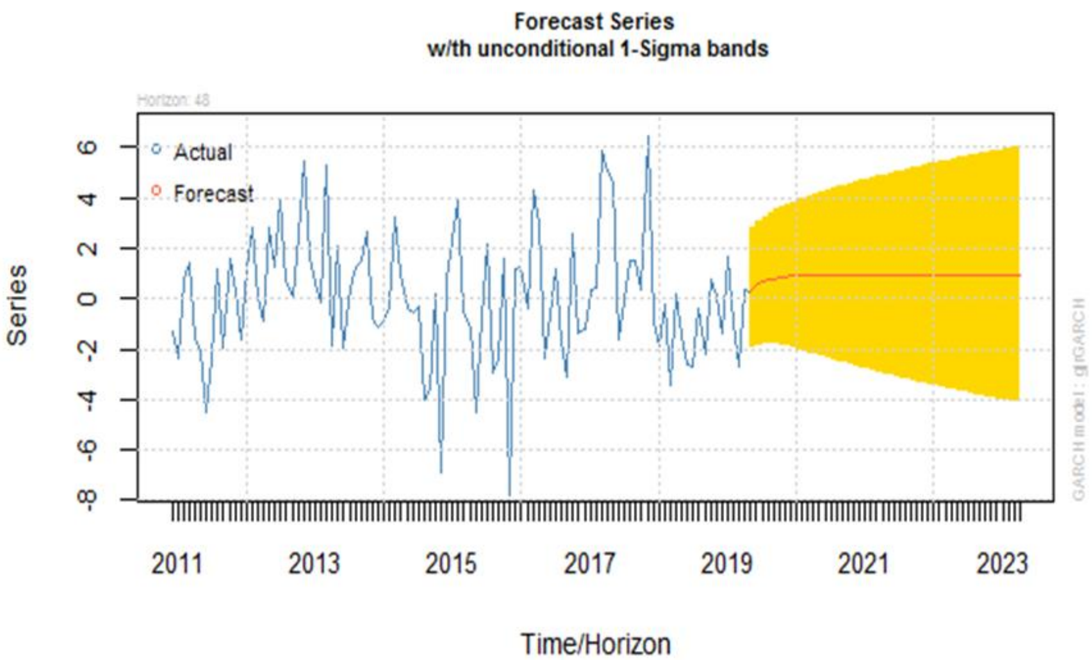

Figure 2: Forecast Plot

The figure 2 is also the forecast plot with unconditional square root of variance, blue line is representing the actual returns series and the red represent the forecast from May, 2019 December, 2023.

4.0 CONCLUSION

This study used the symmetric and asymmetric GARCH models and incorporated them with application of Bai and Perron methodology to detect reasonable structural breaks points in conditional variance using three different statistical distributions for monthly Nigeria stock market return series from Nigeria stock exchange (NSE), from January 1985 to April 2019. After detecting structural breaks, the symmetric GARCH (1,1), the asymmetric GARCH models that is PGARCH (1,1) and GJR-GARCH (1,1) models with considering dummy variables for these structural changes for estimating conditional volatility were selected. The result of GARCH (1,1) with skewed student-t distribution (SSTD), EGARCH (1,1) with student-t distribution (STD), power GARCH (1,1) with skewed student-t distribution (SSTD) and GJR-GARCH (1,1) with skewed student-t distribution (SSTD). After detecting the structural breakpoints in the returns series, the model estimation incorporated structural breaks by including dummy variable in the conditional variance equations of all the models, there was significant reduction of shock persistence parameter and mean reversion rate in all the estimated model. The GJR-GARCH (1,1) model with skewed student-t distribution (SSTD) was found to fit the data better than the others competing models reducing the shock persistence in Nigeria stock market. In conclusion, considering dummy variable for the sudden shock, there is less persistence in variance, meaning a more accurate, moderate and reliable volatility estimate in this regard

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